NOTES ON READING-BASED SEMANTIC NETWORKS FOR NATURAL LANGUAGE

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C O N T E N T S

ABSTRACT

This paper presents a very general type of semantic network within which one can represent the semantic structures of a wide range of natural language constructions in a purely diagrammatic way, and to a degree of articulation sufficient also to support the simultaneous diagrammatic representation of the intuitive entailment relations that hold among sentences incorporating them. In particular, entailment relations turning on the semantic structure of the following constructions can be adequately represented within the proposed type of semantic network: cases, determiners of arbitrary sorts (hence including the ordinary quantifiers), sentential connectives, phrasal connectives, the copula, modal and temporal operators, branching quantifiers, pronominal and proverbal referential constructions within and across sentence boundaries, modifier constructions of arbitrary sorts (phrasal and clausal, adjectival and adverbial, and extensional and intensional). The account given herein of the proposed type of semantic network is mainly restricted to a description of its general diagrammatic character and an attempt to justify the claim that it supports a purely diagrammatic representation of entailment. Selected examples involving various of the above sorts of constructions are used for illustration and include cases, a variety of determiners, sentential connectives, phrasal connectives, and the copula.

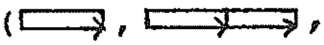
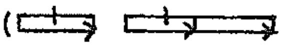
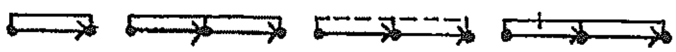
Given a natural language sentence, relative to any given one of its possible readings and relative to fixed but arbitrary

cardinality assumptions regarding the cardinality of denotations, we associate with that sentence a semantic network configuration called an event diagram, which can reasonably be regarded as representing the semantic structure of that sentence (relative to that reading and relative to those cardinality assumptions). A semantic network of the proposed type is defined as a "linked" array of event diagrams, where "links" are dotted lines joining components of different event diagrams. Given, then, a set of natural language sentences, each understood relative to a given reading and taken relative to given cardinality assumptions on denotations, the entailment relationships holding among those sentences are represented within a semantic network as a graphic pattern among their respective event diagrams.

Semantic network configurations such as event diagrams can be regarded either as "abstract" in the sense that geometric figures are abstract, or as "concrete" in the sense that particular drawings or other physical embodiments of those figures are concrete aggregations of, say, ink or graphite particles on a surface, comprising what are sometimes called "tokens" of their associated abstract "types".

Semantic networks as defined herein are built up out of three kinds of basic diagrammatic components: (1) points, (2) braced sequences of arrows called arrow traces, and (3) dotted lines. These have the following graphic character and interpretation:

(1) points (•) represent elements of an underlying (arbitrary) universe of discourse.

(2) arrow traces are "braced" sequences of arrows (
 , ...) and represent relations among elements of the universe of discourse. A bar placed onto the "brace" of an arrow trace () represents the complement of the relation represented by the arrow trace without the bar. Arrow traces are combined with points to form diagrams called arrow paths ( , ...), each of which represents that the elements represented by its constituent points, when taken in the order indicated by the orientation of the arrows in its constituent arrow trace, stand in the relation represented by that arrow trace.

(3) dotted lines (..................) represent the identity relation, and join points to points and arrow traces to arrow traces: a dotted line joining two points represents that those points represent the same element of the underlying universe of discourse; a dotted line joining two arrow traces represents that those arrow traces represent the same relation on elements of the universe of discourse. Dotted lines form linkages within and among event diagrams, forming thereby patterns of interconnections which determine the entailment relations that hold among the sentences represented by those event diagrams.

The event diagrams proposed here are designed to graphically reflect, wholly in terms of the above three types of basic components, a notion of semantic structure developed in an unpublished paper by the author entitled A Theory of Readings (ATR). The ATR notion of semantic structure provides an alternative to predicate logic-based notions of semantic structure, which is very amenable to graphic representation within a network paradigm.

In order to render the graphic structure of our examples fairly transparent and their theoretical description simple, we have employed a uniform diagrammatic paradigm for all event diagrams that is often highly redundant. Such redundancies would ordinarily be eliminated in implementations where processing efficiency was important.

READING-BASED SEMANTIC NETWORKS FOR NATURAL LANGUAGE

O. INTRODUCTORY REMARKS

This paper presents a very general type of semantic network[1] within which one can represent the semantic structures of a wide range of natural language sentences to a degree of articulation sufficient also to support the simultaneous network representation of entailment relations among them.[2,3]

Note 1. By a "semantic network" I mean a diagrammatic figure conceived either as an abstract "type" or as a concrete graphic "token" representing that type that depicts the semantic structure of individual natural language sentences as well as the logical relationships among those structures. A semantic network in this sense is intended as a model of such semantic structures and of their interrelationships that can serve as a conceptual base for possible machine implementations.

Note 2. By "entailment" I mean that relation holding between a set K of sentences on the one hand and a single sentence k on the other, whereby any circumstance under which all the sentences of K are true must also be a circumstance under which k is true as well. Entailment in this sense is a semantic notion, to be distinguished from proof-theoretic not ons embodied in one or another set of inference rules the soundness of which is ultimately established by appeal to an underlying semantic notion of entailment, such as the one proposed here.

Note 3. By the network representation of entailment among natural language sentences I mean a certain diagrammatic relationship (to be described) that holds among the diagrammatic figures repre- senting the semantic structures of those sentences under given readings of those sentences if and only if certain intuitive entailment relationships hold among those sentences under those readings. All such representations are intended to be purely diagrammatic in the sense of containing no labels or other auxiliary linguistic components or descriptions, representing thereby the semantic structure of sentences and of the entailment relations among them purely in terms of network "geometry. We also hope to make the case that these diagrammatic representations afford a compact and economical means of characterizing the complex semantic interconnections among natural language expressions that underlie not only the intuitive entailment relationships into which those expressions enter but other semantic relations as well.

The proposed type of semantic network represents entailment relations in a fully general way, subject only to given cardinality assumptions about the domain of application, but independent of any special lexical assumptions regarding that domain. All meanings of network components are "abstract", by which I mean that networks are built up solely from three basic graphic components each of which designates or, as we shall say, represents, either an element of an underlying (abstract) universe of discourse, a relation among such elements, or the identity relation among such network components signifying that they designate the same element or relation. The semantic structures of natural language sentences and, derivatively, of the entailment relations that hold among them, are thus represented wholly as graphic configurations, that is, diagrams, composed solely of these three basic network components.

Semantic networks of the proposed type are called Reading-Based Semantic Networks (RBSNs). The specific version of RBSNs described herein is based on a particular formal notion of a reading of a natural language word-string that is extremely amen-

Note 3. The semantic structures described herein are also sufficient to support the network representation of various weaker (and more general) forms of entailment which we might refer to as probabilistic entailment, by which I mean any relation holding between a set K of sentences on the one hand and a single sentence k on the other, whereby the probability is p, where $o \leq p \leq 1$, that any circumstance under which all the sentences of K are true is also a circumstance under which k is true as well. The special case of ordinary or "absolute" entailment is obtained then as that case where $p = 1$. To keep the present paper to a reasonable length we do not discuss this notion in this paper beyond the present remark and that of footnote 32 on page 73.

able to representation as a network. That notion is described in detail in an unpublished paper by the author entitled A Theory of Readings (ATR), and serves as a mediating construct between a natural language word-string and its network representation in the sense that that reading provides a set-theoretic description of its semantic structure that is to be diagrammatically represented within an RBSN. By virtue of this connection with readings, all network components of an RBSN are set-theoretically interpretable. In more detail:[4] A reading of a natural language word-string consists of (1) a syntactic[5] representation of that word-string, together with (2) a semantic theory that interprets that syntactic representation in the following sense: it (the semantic theory) specifies the set-theoretic structure of all possible denotations of the syntactic representation, the set of which structures can properly be regarded as the semantic structure of that syntactic representation and, derivatively, as the semantic structure of the word-string itself which that syntactic representation represents. The proposed semantic structures of word-strings are those which appear most adequate to account for intuitively perceived entail-

Note 4. See the Appendix for a summary account of the relationship among natural language word-strings, their readings, and their network representation.

Note 5. A given reading of a given natural language word-string directly involves only such syntactic and semantic factors, and does not directly involve pragmatic factors that condition the choice of appropriate (e.g., dominant) readings of given word-strings relative to given contexts-of-utterance. ATR treats the role of pragmatic factors that condition the choice of readings, including a proposal for assimilating pragmatics within semantics.

ment relationships among a wide and diverse range of natural language word-strings, and the proposed syntactic representations are, in turn, designed to support the simplest possible recursive specifications of those semantic structures.

A reading of a natural language word-string is intended to afford a complete formal specification of a particular way of understanding that word-string.[5.1] Since there are typically multiple possible ways of understanding a given word-string, the choice among which depends on the particular context-of-utterance in which (a token of) that word-string is produced, there are typically multiple possible readings of any given word-string. For simplicity of exposition, we restrict our illustrations of network representations of readings to those readings of word-strings that are "dominant"[6] readings of their respective word-strings in the sense that they formalize the most usual ways of understanding those word-strings across typical contexts-of-utterance.

The RBSNs presented here afford a simplest possible graphic means of representing the semantic structures of arbitrary meaningful natural language word-strings under any of their various possible readings as well as the conditions governing the

Note 5.1. The precise sense of the phrase "a complete . . . word-string" is elaborated in ATR.
Note 6. While, the intuitive meaning of the notion of "dominant" readings is quite clear and will suffice for the general coherence of our presentation, there are important pragmatic issues concerning the proper characterization of this notion, which will not be dealt with here. Some of these issues are addressed in ATR.

entailment relationships that hold among them under those readings.[6.1]

The body of this paper is presented in two sections: In Section 1, we present the basic ideas underlying our approach in terms of an extended discussion of a simple special case. In Section 2, we broaden our discussion to apply to more general cases. An Appendix is also included, in which I attempt to summarize those aspects of the ATR notion of reading that motivate the structure of RBSNs as described in Sections 1 and 2.

Note 6.1. The semantic structures of RBSNs have some parallels with certain aspects of relational databases, but are much more general and flexible than relational databases as presently conceived, particularly in their capacity to represent the semantics of a wide range of diverse natural language constructions, and their consequent capacity to represent natural language entailment among sentences incorporating those constructions.

1. BASIC IDEAS

A universe of discourse is taken as a set of objects, called elements, that one wishes to treat. The underlying logical paradigm[7] adopted in this paper is that all information pertaining to the elements of a universe of discourse can be depicted in terms of whether or not those elements stand in certain relations relative to each other, and that the content of assertions regarding that information can be depicted purely as configurations of diagrammatic representations of elements and relations. The intended sense of what it means for the content of assertions to be so depicted will become clearer in our subsequent examples and discussions.

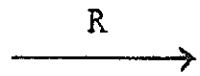
Elements of the universe of discourse are represented as points (•), and relations among those elements are represented as sequences[7.1] of arrows (⟶) and dotted lines (·····). Single arrows are used to represent one- and two-place relations; sequences[7.1] of such arrows are used to represent relations of higher place number. Thus, for example, if a and b are elements of the universe of discourse which are represented[8] respectively by the points:

Note 7. The various assumptions underlying the reasonableness of this paradigm are explored in ATR.

Note 7.1 More exactly, relations among elements are represented as "braced" sequences of arrows, as will be described in Section 2. In the present section we restrict ourselves to simple special cases not requiring braces.

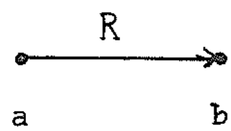
Note 8. The labels "a", "b", "R", and so on that occur in these representations are included for expository purposes only, and do not appear in actual RBSNs.

and if R is a two-place relation on elements of the universe of discourse which is represented by the single arrow:



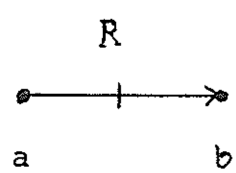
then, if the elements a and b, taken in that order, (that is, the order where a is taken first and b is taken second), stand in the relation R, we represent this state of affairs by the diagram:

(1)

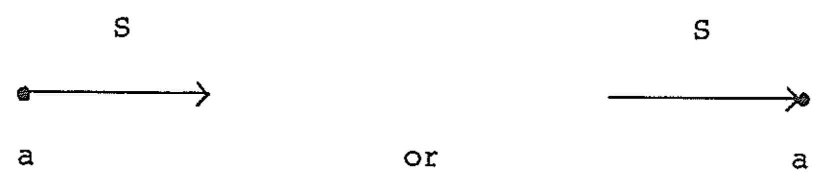


and, if the elements a and b, taken in that order, <u>fail</u> to stand in the relation R, we represent this state of affairs by the diagram:

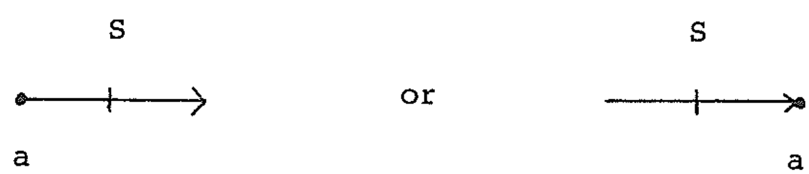
(2)



We also use a single arrow to represent a one-place relation on the elements of the universe of discourse, that is, a "property" of elements of the universe of discourse. Thus if the element a stands in the one-place relation S, that is, has the property S, we represent this state of affairs by the diagram:

(3) 

and, if the element a fails to have the property R, we represent this state of affairs by the diagram:

(4) 

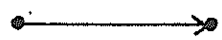
Diagrams of the forms (1) - (4), which are built up out of arrows and points in the manner shown, are called <u>arrow paths</u>. Whether a given single arrow represents a two-place or one-place relation is determined by whether exactly two points or exactly one point occurs with the arrow; thus the arrow in the arrow path (1) represents a two-place relation since exactly two points occur with the arrow, while the arrow in the arrow path (3) represents a one-place relation since exactly one point occurs with the arrow.[8.1]

In the immediately following subsection we describe how a

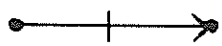
<u>Note 8.1</u> This sort of ambiguity is resolved by attaching "braces" to arrows and arrow sequences, in a manner to be described in Section 2. We use unbraced arrows here in Section 1 and, occasionally, elsewhere, for visual perspicuity.

particular simple sentence whose main verb is represented as a two-place relation is to be represented within an RBSN. This special case exhibits many of the features of our general approach, discussed in Section 2.

1.1 <u>Event Particular Diagrams</u>. By combining simple diagrams of the forms:

(1) 

and

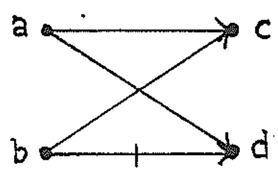
(2) 

we can simultaneously represent, for each pair of a multiplicity of given elements and for a given two-place relation R, that the elements of that pair, taken in a given order, stand in or fail to stand in the relation R. For example, if we have four elements a, b, c, d of the universe of discourse and a two-place relation R, which is such that (i) the elements a, c, taken in that order, stand in the relation R, (ii) the elements a, d, taken in that order, stand in the relation R, (iii) the elements b, c, taken in that order, also stand in the relation R, and (iv) the elements b, d, taken in that order, <u>fail</u> to stand in the relation R[9], we refer to the "circumstance" described by (i)-(iv)

<u>Note 9</u>. We can thus regard diagram (A) (given on next page) as simultaneously representing, in part, that the set of ordered pairs {(a,c), (a,d), (b,c)} is included in the relation R, hence that the set {a,b} is a subset of the first domain of R, and that the set {c,d} is a subset of the second domain of R. This situation is generalizable in the obvious sense to relations of more than two places.

as an <u>event particular</u>, and represent it by the following diagram, called an <u>event particular diagram</u> (EPD):

(A)



We refer to the vertical "columns" of points

a • and • c

b • and • d

as <u>point banks</u> of (A) and, reflecting their relative (left-to-right) order in (A), we refer to the point bank

a •

b •

as "the first point bank" of (A), and we refer to the point bank

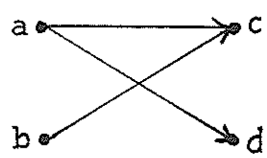
• c

• d

as "the second point bank" of (A).

The case for three-and-higher place relations is wholly analogous. Event particular diagrams, especially when representing relations of more than two places, can become graphically complicated. Thus, generally, in order to simplify event particular diagrams we will often employ the convention of omitting barred arrows (⟶|⟶) from diagrams with the understanding that, when points belonging to successive point banks of a diagram are not joined by any arrow, the elements respectively represented by those points are to be regarded as failing to stand in the relation R. For example, if we apply this convention to the diagram (A) above, (A) can be equivalently represented by the diagram (A'):

(A')



wherein the barred arrow in (A) joining the points representing the elements b,d, in that order, has been omitted.

1.2 Event Diagrams Let us consider an array[9.1] of event

Note 9.1. I use the word "array" in the sense of a graphic configuration of given constituents. There are as many arrays of given constituents as there are ways to arrange them relative to each other. In general, any rearrangement of constituents yields another array though they will be, in an intuitive sense, "equivalent"; for definiteness, we will later distinguish certain arrays as "standard". In this regard, we treat the notion of "array" as more general than that of "diagram" in the sense that diagrams are special sorts of arrays, and regard both diagrams and arrays as linguistic entities akin ontologically to words or sentences. Strictly speaking, one should distinguish between a diagram or array as type and as token. The classical difference between types and tokens is that types are abstract entities of some sort and tokens are concrete, for example, graphic "instances" or "occurrences" of types. It is common practice in talking about words, sentences, and the like to refer, say, to the word "book" ambiguously as to type or token. We follow this particular diagrams (EPDs) (each representing a possible event particular) constructible on a given pair A,B of point banks, and having A as 1st point bank and B as 2nd point bank:

with the understanding that the arrows joining them represent a given relation R, and that a_1, a_2, b_1, b_2 are elements of D represented here by the respective points indicated. We call such an array an <u>event diagram</u> (ED) having those event particular diagrams (EPDs) as <u>constituents</u>. The event diagram represents an <u>event</u>, which is a set of event particulars, that is, a set of possible circumstances in which a_1,a_2 can be related to b_1,b_2 with respect to R. For definiteness, let us assume that the universe of discourse D consists of all human beings, and that the elements a_1,a_2 are all the <u>men</u> in D, and that the elements b_1,b_2 are all the <u>women</u> in D. Let us assume further that the

benign practice in referring to diagrams and arrays. With regard to this practice, we note that, while an array-as-token is properly a configuration of diagrams-as-tokens in the sense that those diagrams-as-tokens occur as physical parts of that array, in which case we refer to those diagrams-as-tokens as <u>constituents</u> of that array-as-token and we can, in a derivative (and somewhat inaccurate sense) also refer to diagrams-as-types as "constituents of" arrays-as-types. Generally, we will speak of relations such as "is a sub-array of", "is a sub-diagram of", "is an instance of", "is an occurrence of", "is a constituent of", "is identical to", "is similar to", "is an ordering among", "is linked by a dotted line to", and so on, which can ambiguously be understood to relate two tokens, to relate two types, or to relate a type to one of its tokens, without making the precise intended sense explicit.

relation[10] R we are concerned to represent is that of <u>loving</u> as it pertains to men loving women.[11] Then A is the point bank representing all men, and B is the point bank representing all women. Accordingly, the set $\mathfrak{A}$ of all possible EPDs on the point banks A, B, taken in that order, is the following:[12]

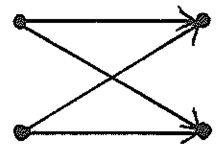
<u>Note 10</u>. We understand "relation" throughout this paper in an <u>extensional</u> sense, whereby the number of such possible relations is wholly determined by the number of point banks, the number of points in each point bank, and the order in which the point banks are to be considered in defining the relation.

<u>Note 11</u>. That is, restricted to the Cartesian Product $\{a_1,a_2\} \times \{b_1,b_2\} = \{(a_1,b_1), (a_1,b_2), (a_2,b_1), (a_2,b_2)\}$.

<u>Note 12</u> We note that there are 16 EPDs in $\mathfrak{A}$. Generally, given the m point banks $A_1,\ldots,A_m$ consisting respectively of $n_1,\ldots,n_m$ elements, there are $2^{n_1 \times \ldots \times n_m}$ possible EPDs with point banks $A_1,\ldots,A_m$, taken in that order. In the case of the set $\mathfrak{A}$, since there were m = 2 sets A and B with 2 elements in each, the total number of possible EPDs with point banks A, B, taken in that order, is $2^{2 \times 2} = 2^4 = 16$.

The set $\mathcal{G}$ of all possible Event Particular Diagrams constructible on the pair of point banks A, B.

A B

(1)


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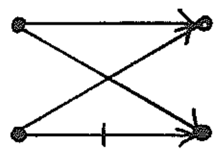

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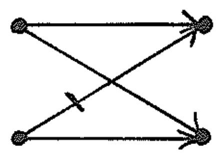

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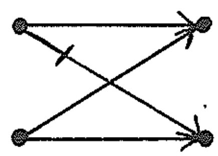

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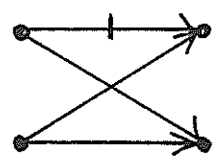

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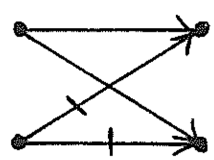

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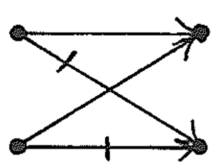

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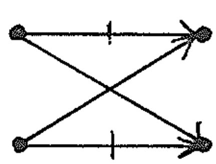

A B

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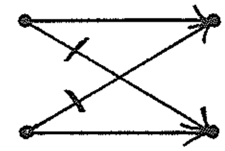

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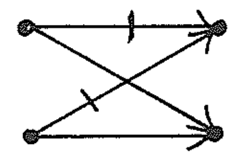

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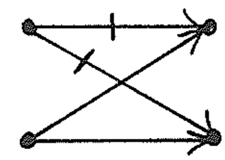

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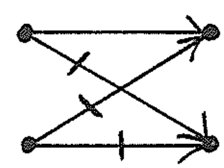

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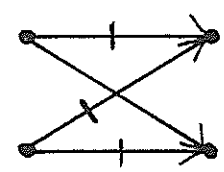

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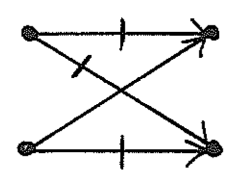

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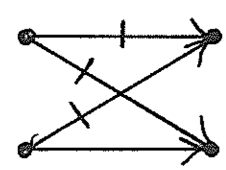
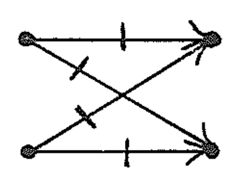


(16)


Under the given meanings of the point banks A, B, and of the arrows joining these points whereby they represent, respectively, the set of men, the set of women, and the (extensional) relation of loving restricted to these two sets as (supersets of) its first and second domains respectively, each of the EPDs of $\mathfrak{A}$ represents a specific circumstance pertaining to them. (For example, EPD(1) represents the circumstance that all men love all women.) An event diagram (ED)[13] is an array of event particular diagrams. The constituents of a given ED are the elements of some one of the $2^{16}-1$ non-empty subsets of $\{(1)-(16)\}$. In this sense an ED represents the set of circumstances represented by its constituent EPDs. (The precise number of men and women in the domain of discourse is unimportant; it is only important that there is a definite number of each.)

1.3 Representing English Sentences as Event Diagrams. We will next indicate how, under these various above assumptions regarding the meanings of the sets A, B, and of the relation between their elements, each of certain English sentences describing men, women, and the relation of loving can be represented by a particular one of the EDs whose constituents are elements of $\mathfrak{A}$ in the sense that the ED representing such a sentence contains as constituents all and only those EPDs of $\mathfrak{A}$ that represent a circumstance under which that sentence would be true. For example, the English sentence "all men love all women" is represented by the ED whose sole constituent is the

Note 13. We will shortly distinguish EDs that are arrays of EPDs as "simple" to distinguish them from the more general case of an event diagram which is an array of arrays of EPDs. This more general case is introduced on page 99.

sole element in the singleton subset $\{(1)\} \subseteq \mathfrak{A}$. In particular, we will identify those EDs the set of whose constituents is among the subsets of $\mathfrak{A}$ which represent given English sentences and, for each ED that represents an English sentence, we will identify a specific English sentence which is represented by it.

More exactly, we construct a certain set $S_{\mathfrak{A}}$ of English sentences and a certain set $\mathfrak{A}^*$ of subsets of $\mathfrak{A}$ such that there is a 1-1 correspondence α between $S_{\mathfrak{A}}$ and $\mathfrak{A}^*$ such that, for all $e \in S_{\mathfrak{A}}$, $\alpha(e)$ consists precisely of those elements of $\mathfrak{A}$ that represent a circumstance under which e would be true. The set $S_{\mathfrak{A}}$ is obtained by (i) filling in each of the blanks of the schema:

(C) _____ men love _____ women

by any ordinary English determiner phrase (e.g., "some", "all", "no", "at least one", "at most one", etc.) to yield what we will call an atomic sentence and (ii) forming successive sentential compounds out of atomic sentences, using the English sentential connectives "and", "or", and "not". Each sentence e of the thus constructed set $S_{\mathfrak{A}}$ corresponds to some non-empty subset $\alpha(e)$ of $\mathfrak{A}$ in the sense that $\alpha(e)$ consists of all and only those EPDs of $\mathfrak{A}$ which represent a possible circumstance which can hold if the sentence e is true. There are $2^{16} - 1 = 65535$ such possible subsets [14] of $\mathfrak{A}$, only some of which correspond to actual

Note 14. In order to verify these claimed correspondences the reader, for the present, will have to rely on an "intuitive reading" of these sentences as well as on an intuitive appraisal of what the EPDs (1)-(16) represent. The formal basis for these correspondences derives jointly from the formalization of the notion of reading, as carried out in ATR and described, at least in part, in the present paper, and on the way that this formalized notion is to be represented within RBSNs, as outlined in the Appendix.

English sentences in this sense. We will identify those subsets that do so correspond to English sentences after we consider some concrete examples involving, first, atomic sentences obtained from schema (C), and second, sentential combinations of such sentences.

Finally, we regard a sentence as corresponding to an ED if that sentence corresponds to the set of constituents of that ED. The ED represents the semantic structure of the sentence to which it corresponds, though occasionally, for grammatical flexibility, we will speak of the ED as "representing the sentence" to mean that the ED represents the semantic structure of that sentence.

The atomic sentence:

(i) All men love some woman

corresponds to an ED whose constituents are the elements of the set {(1),(2),(3),(4),(5),(7),(8),(9),(10)}

The atomic sentence

(ii) Some men love some women

corresponds to the ED whose constituents are the elements of the set {(1),(2),(3),(4),(5),(6),(7),(8),(9),(10),(11),(12),(13),(14),(15)}.

The atomic sentence

(iii) No men love some women

corresponds to an ED whose single constituent is an element of the set {(16)}.

The atomic sentence

(iv) At least one man loves at least two women

corresponds to an ED whose constituents are the elements of the set {(1),(2),(3),(4),(5),(6),(11)}.

The atomic sentence

(v) At least two men love some woman

corresponds to an ED whose constituents are the elements of the set {(1),(2),(3),(4),(5),(7),(8),(9),(10)}.[15]

The atomic sentence

(vi) At least one man loves no woman

corresponds to an ED whose constituents are the elements of the set {(6),(11),(12),(13),(14),(15),(16)}.

The atomic sentence

(vii) Exactly one man loves at least one woman

corresponds to an ED whose constituents are the elements of the set {(6),(11),(12),(13),(14),(15)}.

The atomic sentence

(viii) Exactly one man loves exactly one woman

corresponds to an ED whose constituents are the elements of the set {(12),(13),(14),(15)}.

Note 15. Because of the specific limitation imposed on the sizes of the sets used in this example, namely 2 men and 2 women, this sentence corresponds to the same ED as does the sentence (i).

The atomic sentence

(ix) At most one man loves at most one woman

corresponds to an ED whose constituents are the elements of the set $\{(6),(11),(12),(13),(14),(15),(16)\}$.

We next consider examples involving <u>sentential combinations</u> of atomic sentences. We first make a useful general observation: sentences obtained as sentential combinations of atomic sentences using natural language <u>conjunction</u>, <u>disjunction</u>, and <u>negation</u>, correspond to EDs built up out of the EDs to which their component sentences correspond by the respective set-theoretic operations of <u>intersection</u>, <u>union</u>, and <u>complementation</u> (with respect to $\mathfrak{A}$). That is to say, designating the set of constituents of the ED corresponding to the sentence s by [s], we have:

$$[s_1 \text{ and } s_2] = [s_1] \cap [s_2]$$

$$[s_1 \text{ or } s_2] = [s_1] \cup [s_2]$$

$$[\text{it is false that } s] = \mathfrak{A} - [s]$$ [16]

For example:

The sentence obtained as the <u>conjunction</u> of sentences (iv) and (vi), namely (x) below:

<u>Note 16</u>. These relationships apply only to the special present case where the EPDs operated upon are "similar" in the sense that they have the same point banks and the same relation. For the more general case where the EPDs operated upon are not similar in this sense, these relationships need to be formulated in terms of the more general concepts of Section 2.

(x) At least one man loves at least two women and at least one man loves no woman

corresponds to an ED whose constituents are elements of the set

$[iv] \cap [vi] =$

$\{(1)-(6),(11)\} \cap \{(6),(11)-(16)\} = \{(6),(11)\}$.

The sentence obtained as the <u>disjunction</u> of sentences (iv) and (vi):

(xi) At least one man loves at least two women or at least one man loves no woman

corresponds to an ED whose constituents are elements of the set

$[iv] \cup [vi] =$

$\{(1)-(6),(11)\} \cup \{(6),(11)-(16)\} = \{(1)-(6),(11)-(16)\}$.

The sentence obtained as the <u>negation</u> of sentence (iv):

(xii) It is false that at least one man loves at least two women (i.e., no man loves at least two women)

corresponds to an ED whose constituents are elements of the set

$\mathfrak{A} - [iv] = \mathfrak{A} - \{(1)-(6),(11)\} = \{(7)-(10),(12)-(16)\}$.

The sets [s] formed in this way comprise an algebra on $\mathfrak{A}$ under the operations of intersection, union, and complementation.

It is of interest to note that we can characterize the elements of this algebra on $\mathfrak{A}$ independently of any reference to the sentences to which they correspond, namely as the set of non-empty subsets of $\mathfrak{A}$ whose elements (i.e., EPDs) have a certain simple structural relationship to each other.[17] We can describe the requisite structural relationship in a completely general way as follows: Let $B_1,\ldots,B_m$ be m point banks, and let $\mathcal{L}$ be the set of all relations holding on the points of $B_1,\ldots,B_m$, taken in that order. Let EPDs $e_1,e_2 \in \mathcal{L}$, and let α be a one-to-one function on the point banks $B_1,\ldots,B_m$ of $\mathcal{L}$ such that: (1) for each $1 \leq i \leq m$, α maps the points of B_i onto the points of B_i, and (2) for all points b_1 of $B_1,\ldots,b_m$ of B_m, $b_1,\ldots,b_m$ are joined by unbarred (barred) arrows in e_1 if and only if $\alpha(b_1),\ldots, \alpha(b_m)$ are joined by unbarred (barred) arrows in e_2. If there is such a function α relating e_1 and e_2, then e_2 is called a rearrangement of e_1, e_1 is called a rearrangement of e_2, and α is called a rearrangement function on $\mathcal{L}$. A subset $\mathcal{L}'$ of $\mathcal{L}$ is said to be closed under rearrangements if and only if for all $e_1,e_2 \in \mathcal{L}$, if $e_1 \in \mathcal{L}'$, and e_2 is a rearrangement of e_1, then $e_2 \in \mathcal{L}'$. We state the following without proof: Let $\mathcal{L}$ be the set of all EPDs on point banks $B_1,\ldots,B_m$, let D be a schema analogous to schema C but containing m common-noun phrases instead of two (e.g., men and women) and an m-place relation instead of a two-place relation (e.g., Love), and let $S_{\mathcal{L}}$ be the set of all sentences obtained from schema D by the same procedure (see page 15) as the set $S_{\mathfrak{A}}$ is obtained from schema C. Then, for every

Note 17. This structural relationship, as it pertains to the set $\mathfrak{A}$ of EPDs (1) - (16), intuitively reflects the fact that those EPDs cannot be distinguished by English sentences involving only reference to men, women, and loving (as it pertains to men loving women), restricted to the means prescribed here.

non-empty subset $\mathcal{B}'$ of $\mathcal{B}$,

there is an English sentence corresponding to $\mathcal{B}'$ if and only is $\mathcal{B}'$ is closed under rearrangements. For the case where $\mathcal{B}$ is $\mathcal{A}$ as described earlier, this claim specializes as follows: for every non-empty subset $\mathcal{A}'$ of $\mathcal{A}$, there is an English sentence corresponding to $\mathcal{A}'$ if and only if $\mathcal{A}'$ is closed under rearrangements; moreover, the set of such English sentences is the set of all sentential combinations of sentences (C') obtainable from (C) by filling in the blanks of (C) by determiner phrases.

We note that the relation "being a rearrangement of" is an equivalence relation on $\mathcal{B}$ (i.e., it is a reflexive, symmetric, transitive relation on $\mathcal{B}$), hence induces a partition of $\mathcal{B}$ into a set of disjoint cells $\mathcal{B}^* = \{\mathcal{B}_1, \ldots, \mathcal{B}_k\}$ such that, for each $1 \leq i \leq k$, $\mathcal{B}_i$ consists of all and only EPDs $b \in \mathcal{B}$ that are rearrangements of each other. In the case of the set $\mathcal{A}$, there are 7 cells in $\mathcal{A}^*$, namely: {(1)}, {(2),(3),(4),(5)}, {(6),(11)}, {(7),(10)}, {(8),(9)}, {(12),(13),(14),(15)}, and {(16)}; consequently, there are $2^7-1 = 127$ subsets of $\mathcal{A}$ that are closed under rearrangements. This means that (only) 127 of the possible $2^{16}-1 = 65,535$ non-empty subsets of $\mathcal{A}$ have English sentences that correspond to them (or sentences of any natural language, for that matter). This claim is based on the following observation: There are exactly 127 non-empty subsets of $\mathcal{A}$ that can be obtained as iterated unions of the above 7 cells, each such iterated union corresponding to that non-empty subset of $\mathcal{A}^*$ consisting of all the cells that enter into that union, and there are exactly 127 non-empty subsets of $\mathcal{A}^*$. Sample[18] sentences

Note 18: These sentences are obviously not unique.

corresponding to the above 7 cells (EDs) are respectively as follows:

the sentence

(xii) All men love all women

corresponds to an ED whose constituents are elements of the set {(1)};

the sentence

(xiii) Some man loves all women and some man loves exactly one woman

corresponds to an ED whose constituents are elements of the set {(2),(3),(4),(5)};

the sentence

(xiv) Exactly one man loves all women

corresponds to an ED whose constituents are elements of the set {(6),(11)};

the sentence

(xv) No man loves all women

corresponds to an ED whose constituents are elements of the set {(7),(10)};

the sentence

(xvi) All men love exactly one woman

corresponds to an ED whose constituents are elements of the set {(8),(9)};

the earlier sentence

(viii) Exactly one man loves exactly one woman

corresponds to an ED whose constituents are elements of the set {(12),(13),(14),(15)};

the earlier sentence

(iii) No men love some woman

corresponds to an ED whose constituents are elements of the set {(16)}

Thus every sentence constructed as a recursive sentential combination of sentences of the form (*C*) is, under the cardinality assumption that there are exactly two men and two women, equivalent to some disjunction of sentences (iii), (viii), (xii), (xiii), (xiv), (xv), and (xvi). We note that this holds also for further sorts of sentences such as the following, which involve restrictive relative clauses:

All men who love any women love every woman

which corresponds to an ED whose constituents are elements of the set {(1),(6),(11),(16)} and which is equivalent to the sentence:

All men who fail to love any woman fail to love every woman

hence which also corresponds to an ED whose constituents are elements of the set {(1),(6),(11),(16)}.

Each of these latter two sentences is equivalent to the disjunction of the three following "basic" sentences:

(a) All men love all women

which corresponds to an ED whose constituents are elements of the set $\{(1)\}$,

(b) Exactly one man loves all women

which corresponds to an ED whose constituents are elements of the set $\{(6),(11)\}$,

and

(c) No man loves any woman

which corresponds to an ED whose constituents are elements of the set $\{(16)\}$. These three sentences can be combined by the sentential connective "or" to form the sentence

All men love all women or exactly one man loves all women or no man loves any woman

which corresponds to an ED whose constituents are elements of the set $\{(1),(6),(11),(16)\}$.[19]

1.4 <u>Representing Entailment Relationships Among English Sentences</u> In the above examples, all EPDs were of a restricted kind, namely (i) they all had the same point banks,

<u>Note 19</u>. The claimed reasonableness of the correspondences mentioned above between English sentences and subsets of $\mathcal{G}$ is intuitively verifiable by "reading" the English sentence in question and "applying" that reading to each EPD of $\mathcal{G}$ to determine whether that EPD actually "depicts a circumstance" under which that sentence would be regarded as true. The subset of $\mathcal{G}$ which corresponds to that sentence would then consist of all and only those EPDs of $\mathcal{G}$ which so depict a circumstance under which the sentence in question would be regarded as true.

This intuitive basis for identifying the appropriate correspondences between natural language sentences and EPDs can be supplanted by a mechanical procedure, described in ATR, which associates the requisite EPDs of any given natural language sentence with that sentence which has been assigned a reading. The related task of developing mechanical procedures for obtaining suitable readings of natural language sentences (i.e., the "parsing" task) has only been preliminarily addressed.

(ii) their constituent arrows were all "pointed" in the same direction, namely from points of the point bank representing men to points of the point bank representing women, and (iii) their constituent arrows represented the same relation, namely that of loving as it pertains to men loving women. We shall refer to EPDs that are related in this way as similar[20] to each other.

With reference to the above set of examples involving similar EPDs, we note that one sentence s entails another sentence t just in case the set [s] corresponding to s is set-theoretically included in the set [t] corresponding to t. Under the cardinality assumptions expressed by the diagrams of $\mathfrak{A}$, namely that there are exactly two men and exactly two women, we conclude (by virtue of this set-theoretic inclusion criterion for entailment) that, for example, the following hold: (i) entails (ii) and (v), (i) and (v) inter-entail each other, and (vii) entails (vi) and (ix); also (vi) entails (ix), (viii) entails (ix), (iv) entails (xi), and (x) entails both (iv) and (xi). We further note that any two sentences from among the above are contradictories of each other just in case their corresponding sets are set-theoretic complements of each other with respect to $\mathfrak{A}$; accordingly, (ii) and (iii) are contradictories, as are (iv) and (xii). Thus entailment, for the present special case, where the EPDs considered are all similar[21], can be represented within RBSNs in terms of simple set-theoretic inclusion. The diagrammatic representation of general cases of entailment encompassing entailment relations among non-similar EPDs as well requires a

Note 20. This notion is defined more precisely later. See pages 61-63.
Note 21. See pages 73 and 74.

more general characterization of entailment among sets than set-theoretic inclusion. Such a more general characterization will be given at the end of Section 2 in terms of the existence of certain pathways that link the constituent points and arrows of the entailing and entailed EDs. In order to accommodate the intended generalization as well as to extend its application to relations of an arbitrary finite number m of places with arbitrary m-tuples of domains, we will need to extend various of the preceding notions; this will be done in Section 2 below.

We note that in the above examples, the entailment relationships did not depend in any way on the particular meanings of the points or arrows in the event particular diagrams of $\mathcal{A}$ (as men, women, and loving) that we have assigned to them in those diagrams, but depended only on the structural relationships among those diagrams. This derives from the fact that the structure of the ED corresponding to any given sentence depends only on the meanings of the logical words, such as all, some, and, not, etc., occurring in that sentence, and on the cardinality assumptions that there are exactly two men and two women, and not on the meaning of the lexical words, namely men, women, and love, occurring in it, so that the structural relationships among EDs depend only on the meanings of the logical words in their corresponding sentences. (The cardinality assumptions limit only the range of possible EPDs in $\mathcal{A}$, but not the structural relationships among EDs of $\mathcal{A}$).

In this sense, entailment relationships among sentences imposed by the structural relationships among their corresponding EDs are completely general, except to the extent that the cardinality assumptions may force two intuitively inequivalent sentences[22] whose logical content is directly impinged upon by those cardinality assumptions to have the same corresponding EDs.

The relationship between the present paper and ATR is that ATR provides ways of constructing readings for a very wide range of natural language sentences and, given a natural language sentence with an associated reading of the type proposed in ATR, one can construct an ED corresponding to that sentence whose structure is determined by that reading. In the present paper we do not discuss this relationship in a general way nor do we exhibit actual readings of given natural language word strings. We instead discuss only those aspects of this relationship which directly affect network representations of entailment relationships among the English sentences we use here for illustration, and allow that the intended readings of these sentences are intuitively ascertainable without recourse to their explicit presentation. In this section we have presented some of the basic ideas underlying network representation of the semantic structure of natural language sentences in terms of a simple special case. In Section 2 below we will extend this discussion to cover more general cases.

Note 22. For examples of this, see page 17 and Note 15.

2. EXTENSION OF THE BASIC IDEAS

In this section we extend our considerations of the preceding section in order to accommodate the network representation of the semantic structure of much more general types of natural language sentences and of entailment relations among them. In order to accomplish this we need to extend our notion of "arrow path" and of "event particular diagram", and to formulate entailment conditions on event diagrams representing sentences so that they apply also to these extended notions.

2.2 Arrows and Braces. Our concern in this subsection is to informally describe and illustrate the use of arrow paths that contain an arbitrary finite[23] number of arrows. In the following Section 2.2, we will describe the notions more precisely and assign suitable set-theoretic interpretations to them. Our interest is to provide diagrammatic representations of sentences[24] like the following under any of their possible readings which are sufficiently articulated to support the specification of suitable set theoretic interpretations for these diagrammatic representations to yield all the intuitively reasonable entailments that hold among them under those given readings.

Note 23. The imposition of finiteness is clearly unnecessary for the general theory, for all our descriptions could be generalized to accommodate infinite-place relations and/or infinite domains, but finiteness is necessary for our intended physical interpretations.

Note 24 While these sentences illustrate some of the required generalizations of the arrow path structures we are discussing, they have specially simple quantificational structures, wherein each noun phrase designates a single element. More general quantificational structures will be illustrated later.

1) John gave the book to Mary.

2) John did not give the book to Mary.

3) John gave the book.

4) The book was given to Mary.

5) John did not give the book.

6) The book was not given to Mary.

7) John gave the book and the book was given to Mary.

8) John did not give the book and the book was not given to Mary.

9) John gave the book and the book was not given to Mary.

10) John did not give the book and the book was given to Mary.

11) It is false that John gave the book and the book was given to Mary.

12) John gave to Mary.

13) John did not give to Mary.

14) It is false that John gave the book and the book was given to Mary

15) It is false that John did not give the book and the book was given to Mary.

16) It is false that John did not give the book and the book was not given to Mary.

17) John did not give the book but gave to Mary.

18) John gave the book but not to Mary.

19) It is false that John gave the book but not to Mary.

20) It is false that John gave to Mary but the book was not given to Mary.

21) John gave the book but the book was not given to Mary.

22) It is false that John did not give the book but gave to Mary.

23) John gave

24) John did not give

25) The book was given

26) The book was not given

27) Mary was given to

28) Mary was not given to

29) John gave and Mary was given to

30) John did not give and Mary was given to

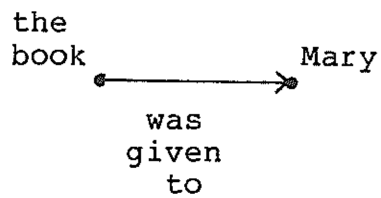
31) John gave and Mary was not given to

32) John did not give and Mary was not given to

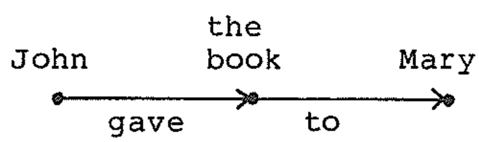
We will first informally examine some of the issues involved in the proper diagrammatic representation of the above sentences and introduce suitable extensions of our earlier notions required to handle them. Later, in the following Section 2.2, we will formulate general definitions of these extended notions required for the treatment of the general case.

The binary relations gave and was given to can be represented by single arrows, in the manner already discussed in Section 1. Accordingly, (3) and (4) can be represented respectively by the arrow paths[25] (3') and (4'):

(3') 

(4') 

and (7), which is a "combination" of sorts of (3) and (4), can be represented by (7') whose diagrammatic[25] construction out of (3') and (4') parallels, in the obvious sense, the syntactic construction of (7) out of (3) and (4):

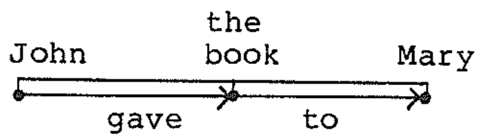
(7') 

Since (1) differs from (7) we require a representation (1') of (1) that is graphically different from the representation (7') of (7). Moreover, the representations (1') and (7') must differ graphically in ways that are set-theoretically interpretable in a coherent way. The key intuitive difference

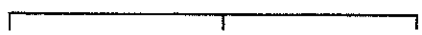
25 We label the points and arrows for convenience in reference. Such labels, of course, do not officially occur in arrow paths. The identity of corresponding points and arrows is diagrammatically expressed by dotted lines, as described in Section 2.2 below.

between (7) and (1), of course, is that, in (7), the act of John giving the book may not have included Mary as the entity to whom that book was given , whereas in (1), the act of John giving the book indeed does include Mary as the entity to whom the book was given. In order to represent that this latter is indeed the case, i.e., to represent (1'), we impose a *brace* onto the arrow path (7') to diagrammatically represent (1) as (1'):

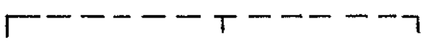
(1')



We call the brace:

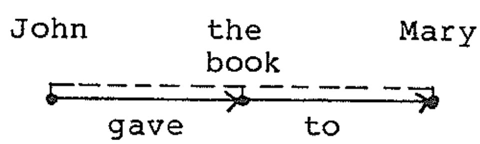


occurring in (1) a *strong brace*, and distinguish it from a *weak brace*:



which can also be applied to (7') to yield:

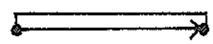
(1'')



,

and whose set-theoretic interpretation is the same as that of (7').[25.1]

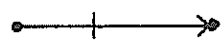
We introduce some simplifying conventions: By virtue of the set-theoretic interpretations[25.1] of strong and weak braces, the arrow paths containing a single arrow:



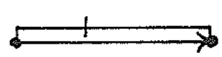
and

are interpreted similarly. Because of this, we will often abbreviate both the above arrow paths alike by the simpler:

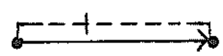
Similarly



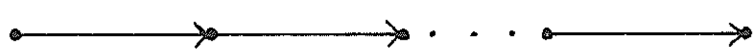
is to be regarded as a diagrammatic abbreviation for



as well as for

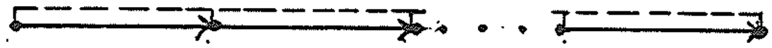


For simplicity of graphic representation we will also, in general, interpret an arrow path containing a sequence of n arrows without a brace



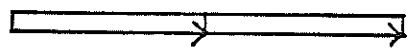
Note 25.1. For these interpretations, see Section 2.2 below, especially page 47, and Note 27.1.

as if it occurred with a weak brace[26.1]



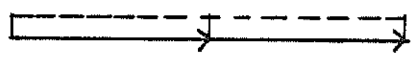
We note that diagram (1') above is composed of two types of components, namely three points and the graphic figure

(1.1')



which is composed, in turn, of a strong brace and a sequence of two arrows. In a wholly analogous way, diagram (1'') above is composed of three points imposed onto the graphic figure

(1.1'')

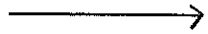


which latter is composed, in turn, of a weak brace and a sequence of two arrows. Figures of the forms (1.1') and (1.1'') are called <u>arrow traces</u>; more finely, (1.1'), since it contains a strong brace, is called a <u>strong arrow trace</u>, and (1.1''), since it contains a weak brace, is called a <u>weak arrow trace</u>.

The utilization of strong and weak braces enables us, not only to graphically express the above noted differences in the meanings of sentences (1) and (7), but is also essential for representing <u>complements</u> of representable relations. We earlier noted that we represent the complement of the relation represented by the diagram:

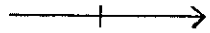
<u>Note 26.1.</u> While weak braces are not essential in the immediately preceding examples, they are essential in our general treatment, proving a richer means of diagrammatic expression and a greater coherence in our account of the structure of diagrams than could be attained without them. Some evidence of their utility is given below in this section, and later in Section 2.7.

(a)



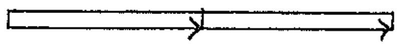
by placing a bar "|" across the arrow shaft of (a), to yield the following diagram:

(a')



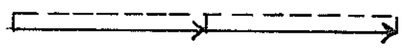
In a wholly analogous way we represent the complements of the relations represented by the diagrams:

(b)



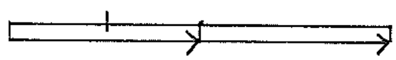
and

(c)



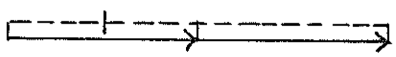
by placing a bar across the braces of (b) and (c) to form the diagrams:

(b')



and

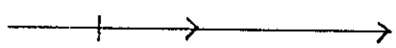
(c')



respectively.

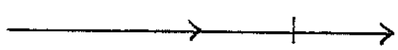
Note that the use of braces (or a similar graphic device) is essential for this purpose insofar as the placing of a bar across an arrow, as in

(d)

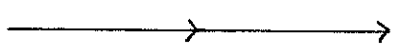


or

(e)



would not represent the complement of the relation represented by the entire unbarred arrow sequence

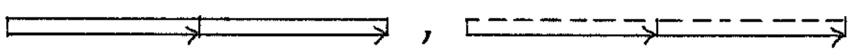


but would rather represent two different relations: the relation represented by (d) would be compounded (see page 47 for the general definition of the intended compound relation) out of the complement of the relation represented by the unbarred version of the left arrow of (d) and the relation represented by the right arrow of (d); the relation represented by (e) would be compounded out of the relation represented by the unbarred version of the left arrow of (e) and the complement of the relation represented by the unbarred version of the right arrow of (e).

Accordingly, we express complementation by regarding the bar representing complementation as applying to the <u>entire</u> arrow trace that is, we adopt the convention that a bar crossing any part of the base of a given brace is regarded as governing the entire brace. For example

mean the same as

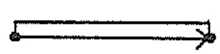
respectively, and represent the complements of the relations represented by



respectively.

In this regard, we note that we can apply our convention to interpret the arrow path

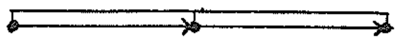
as a graphic abbreviation for the arrow path



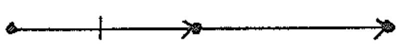
to also interpret the arrow path

as a graphic abbreviation for the arrow path

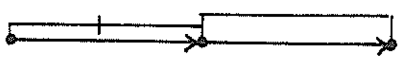
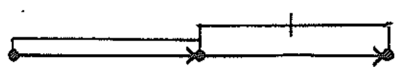
where we draw the braces at different levels to graphically distinguish this case from that of the arrow path



which is interpreted differently. Analogously, we interpret the arrow paths

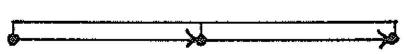
 , 

as graphic abbreviations for the arrow paths

 , 

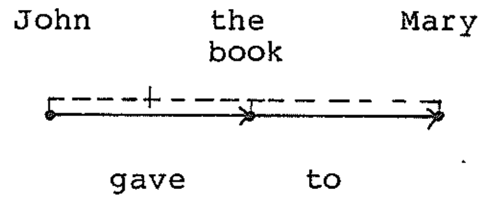
which, by virtue of the immediately preceding discussion, are each interpreted differently from

which represents the complement of the relation represented by

 .

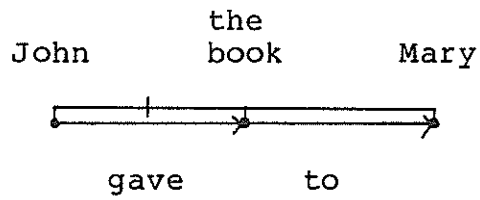
Thus, we would express the negation of (7), namely (11) by

(11')



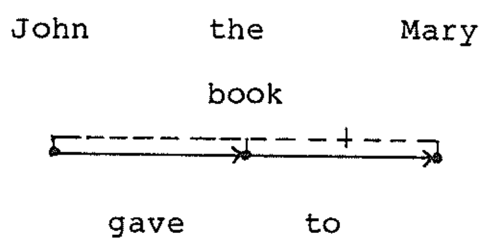
and the negation of (1) namely (2) by

(2')

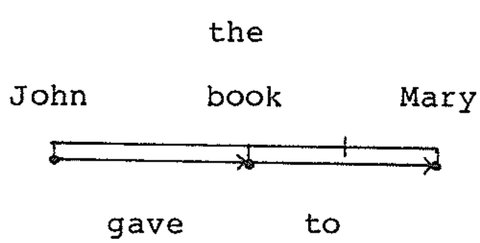


We recall that the precise location of a bar "|" occurring on the brace of an arrow trace t is unimportant, so that we could as well have placed the bar in (11') and (2') over the second arrow to yield the equivalent arrow paths:

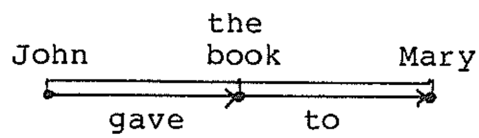
(11")



(2")

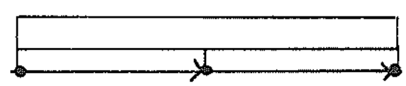


Braces also enable us to interpret <u>non-contiguous</u> subsequences of given arrow paths: thus, for example, a non-contiguous subsequence of the arrow path;

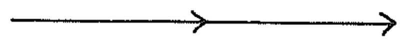
(1') 

would be:

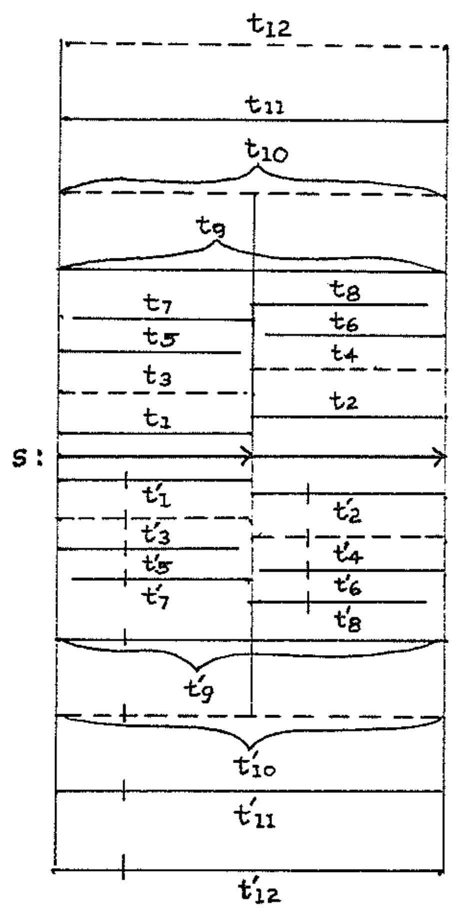
which we could indicate by superposition onto (1') as in:



More generally, we can distinguish among the various arrow traces definable on a given sequence s of arrows by drawing each such arrow trace at a <u>different height off the sequence s</u>. For example, we draw the possible arrow traces definable on the sequence

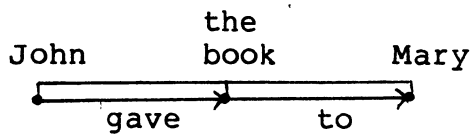
s: 

of arrows as:

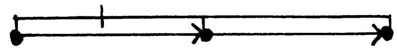


The ultimate utility of graphically distinct diagrams, of course, resides in their intended set-theoretic interpretations, which will be shortly described in Section 2.2 below.

With the aid of the preceding informal remarks on the intended interpretations of arrow paths, we list below the arrow paths (1')-(28') that represent, respectively, the semantic structures of the sentences (1) - (28) given earlier.

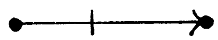
(1') 

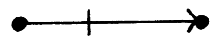
(John gave the book to Mary)

(2')  (John did not give the book to Mary)

(3')  (John gave the book)

(4')  (The book was given to Mary)

(5')  John did not give the book)

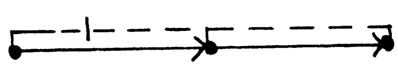
(6')  The book was not given to Mary)

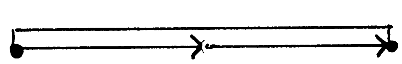
(7')  (John gave the book and the book was given to Mary)

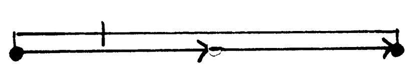
(8')  (John did not give the book and the book was not given to Mary)

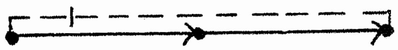
(9')  (John gave the book and the book was not given to Mary)

(10')  (John did not give the book and the book was given to Mary)

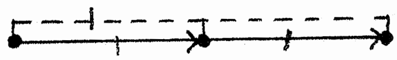
(11')  (It is false that John gave the book and the book was given to Mary)

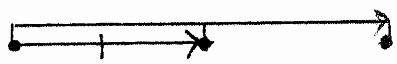
(12')  (John gave to Mary)

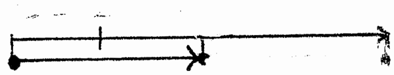
(13')  (John did not give to Mary)

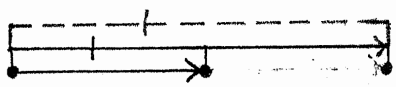
(14')  (It is false that John gave the book and the book was given to Mary)

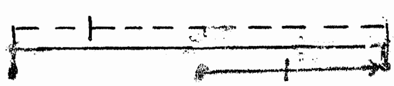
(15')  (It is false that John did not give the book and the book was given to Mary)

(16')  (It is false that John did not give the book and the book was not given to Mary)

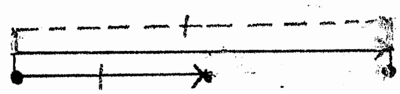
(17')  (John did not give the book but gave to Mary)

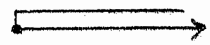
(18')  (John gave the book but not to Mary)

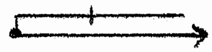
(19')  (It is false that John gave the book but not to Mary)

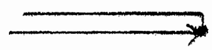
(20')  (It is false that John gave to Mary but the book was not given to Mary)

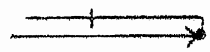
(21')  (John gave the book but the book was not given to Mary)

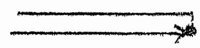
(22')  (It is false that John did not give the book but gave to Mary)

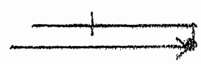
(23')  (John gave)

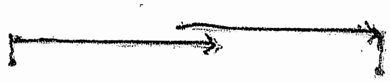
(24')  (John did not give)

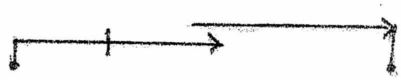
(25')  (The book was given)

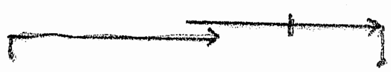
(26')  (The book was not given)

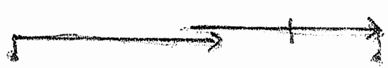
(27')  (Mary was given to)

(28')  (Mary was not given to)[26.2, 26.3]

(29')  (John gave and Mary was given to)

(30')  (John did not give and Mary was given to)

(31')  John gave and Mary was not given to)

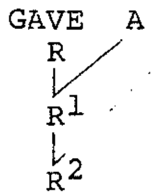
(32')  (John did not give and Mary was not given to)

26.4

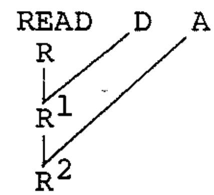
Note 26.2 We will shortly introduce dotted lines to join points intended to represent the identical element of the universe of discourse and to join arrow traces intended to represent identical relations on that universe of discourse. Since dotted lines have not yet been officially introduced for this purpose, (see Sections 2.4, 2.5, below), we employ the temporary expository device of vertically aligning those points and arrow traces that are intended to be similarly interpreted.

Note 26.3 We note that there is no natural English phrasing to express that property possessed by those objects that are given to something (though one could possibly force a phrasing such as "being given from"). Thus we do not include the "missing" arrow paths that would have the same graphic relationship to (27') and (28') respectively that (23') and (24') have to (25') and (26') respectively.

Note 2.6.4 : There are also further kinds of constructions related to the above that we do not attempt to diagrammatically represent in the present paper since their proper representation would involve also an account of the representation of referential links and of clauses, which we do not discuss here. These constructions would include cases where the base relations represented by successive arrows were <u>different</u>, as would be required to diagram the following syntactic representation which has two different base relations:

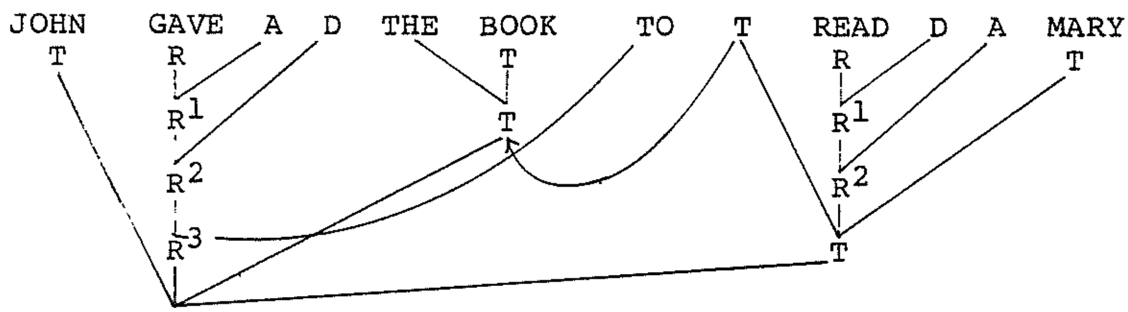


and

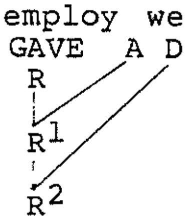


(1) John gave the book to be read by Mary.

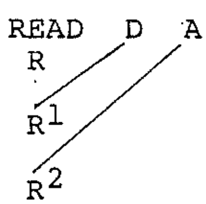
(1')



The means for diagrammatically representing (1') would be, very roughly, to employ weakly braced arrow paths whose first arrow represented



and whose second arrow represented



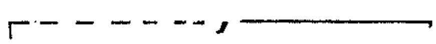
as in:

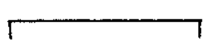
2.2 Arrow Paths and Their Set-Theoretic Interpretation.

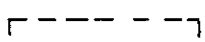
Braces, as illustrated in the previous subsection, are defined, generally, as follows:

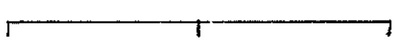
An m-pronged unbarred strong (weak) brace b is a graphic figure composed of a solid (dashed) line, which may be straight or curved, called the base line of b, and m shorter mutually non-intersecting straight lines issuing from the base line, and called the prongs of b. In the special case that m = 1, b consists simply of a single prong without a base line.

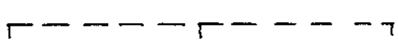
The following are examples of unbarred braces:


1-pronged (weak and strong) unbarred braces


2-pronged strong unbarred brace

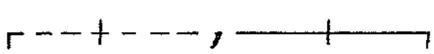

2-pronged weak unbarred brace


3-pronged strong unbarred brace

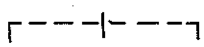

3-pronged weak unbarred brace

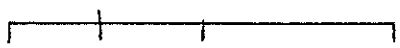
An m-pronged barred (weak) strong brace b^c is an m-pronged unbarred (weak) strong brace b together with a bar (|) which is a short straight line crossing the base line of b perpendicularly at some point.

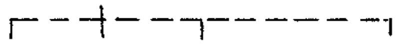
The following are examples of barred braces:


1-pronged weak and strong barred braces

 2-pronged strong barred brace

 2-pronged weak barred brace

 3-pronged strong barred brace

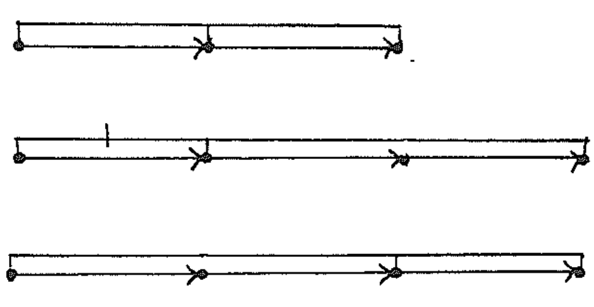
 3-pronged weak barred brace

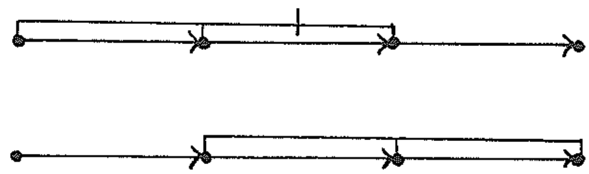
An m-place *arrow path* p is then defined as a sequence of alternating points and arrows together with an m-place unbarred or barred, strong or weak brace all of whose prongs terminate at points of p in such a way that distinct prongs terminate at distinct points of p. p is said to be *strong* or *weak* according as its brace is strong or weak, and *unbarred* or *barred* according as its brace is unbarred or barred.

The *point trace of p* is the m-tuple of points of p at which the prongs of the brace of p terminate.

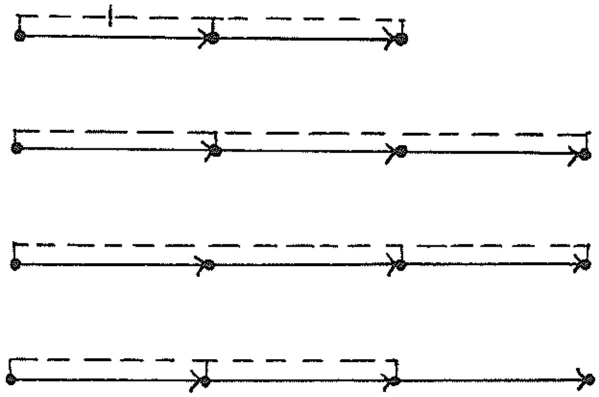
The *arrow trace of p* is the braced sequence of arrows underlying p.

The following are examples of 3-place *strong* arrow paths:



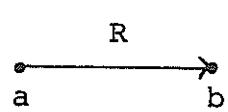


and the following are examples of 3-place *weak* arrow paths:

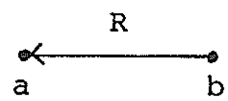


Interpretations of Arrow Paths. The arrow trace of an arrow path p of length m represents an m-place relation on the universe of discourse. The arrow path p itself represents that the elements represented by the successive points of its point trace stand in the relation represented by its arrow trace.

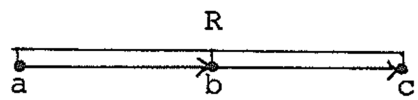
The *order* in which given elements of the universe of discourse are to be considered with respect to a given relation R, say, is represented by the order in which the arrows representing R join the points representing those elements. Thus for example, the diagram

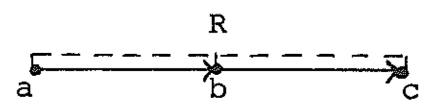


represents that the elements a,b, taken in that order, "stand in" the two-place relation R, while the diagram

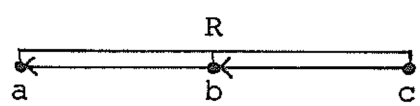


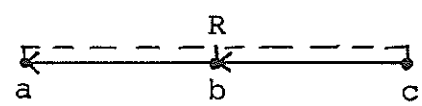
represents that the elements b,a, taken in that order, "stand in the two-place relation R. Similarly, the diagrams



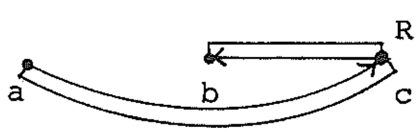


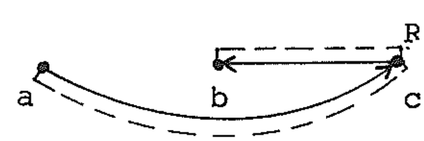
represent that the elements a,b,c, taken in that order, "stand in" the three-place relation R, while





represent that the elements c,b,a, taken in that order, "stand in" the three-place relation R, and the diagram[27]





Note 27. The brace here is "twisted" to the exhibited form to accommodate the orientation of the arrows it braces together.

represents that the elements a, c, b, taken in that order, "stand in" the three-place relation R, and so on.

In order to describe the relationship between the relations that strong and weak braced arrow traces represent and the relations represented by the constituent components of those braced arrow traces, we introduce the following notions:

(a) Let $R_1,\ldots,R_k$ be relations of $n_1,\ldots,n_k$ places on the universe of discourse D, respectively. Then <u>the overlap product of $R_1,\ldots,R_k$</u>, in symbols, $R_1 \otimes \ldots \otimes R_k$, is defined as the relation:

$$\{(a_{11},\ldots,a_{1n_1}, a_{22},\ldots, a_{2n_2}, a_{32},\ldots, a_{3n_3},\ldots, a_{k2},\ldots, a_{kn_k}) \mid (a_{i1},\ldots, a_{in_i}) \in R_i, \text{ for } 1 \leq i \leq n, \text{ and } a_{jn_j} = a_{j+1,1}, \text{ for } 1 \leq j < k\}$$

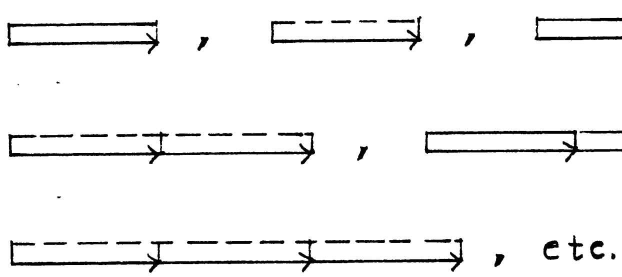
and the <u>adjacent product of $R_1,\ldots,R_k$</u>, in symbols, $R_1 \dot{X} \ldots \dot{X} R_1$, is defined as the relation:

$$\{(a_{11},\ldots,a_{1n_1}, a_{21},\ldots,a_{2n_2}, a_{31},\ldots,a_{3n_3},\ldots, a_{k1},\ldots,a_{kn_k}) \mid (a_{i1},\ldots,a_{in_i}) \in R_i, \text{ for } 1 \leq i \leq n\}$$

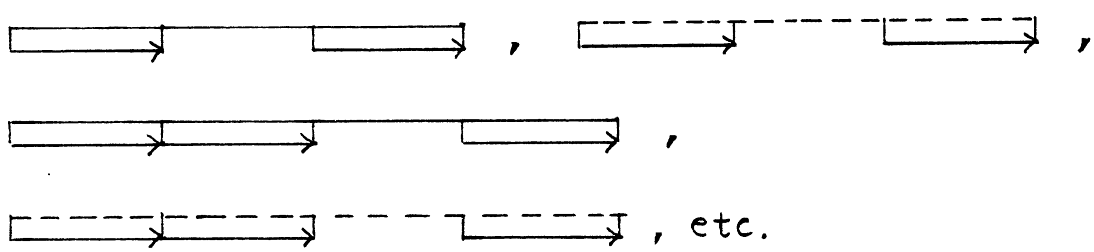
(b) Let R be an m-place relation on the universe of discourse D. Let $1 \leq j_1 < j_2 < \ldots < j_k < m$. Then <u>the restriction of R to the j_1st, j_2nd,..., j_kth domains of R</u> is the relation $R_{j_1,\ldots,j_k} = \{(a_j, \ldots, a_j) \in D^k \mid$ there are $a_1,\ldots,a_{j_1-1}, a_{j_1+1},\ldots, a_{j_2-1}, a_{j_2+1},\ldots, a_{j_k-1}, a_{j_k+1},\ldots, a_m \in D$ such that $(a_1,\ldots,a_m) \in R\}$

Strong and weak braces can be applied to *non-contiguous* as well as contiguous sequences of arrows:

(i) application of strong and weak braces to *contiguous* sequences ofarrows, as illustated earlier:

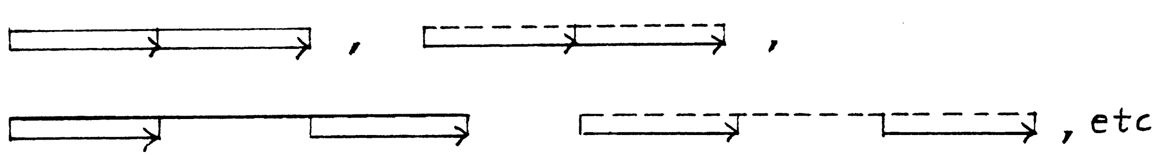


(ii) application of strong and weak braces to *non-contiguous* sequences of arrows:

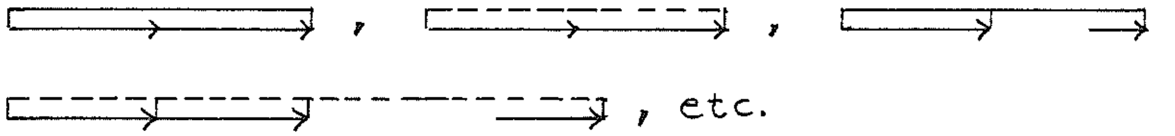


Moreover, strong and weak braces can be applied to non-successive arrow heads and tails:

(iii) application of strong and weak braces to *successive* arrow heads and tails:

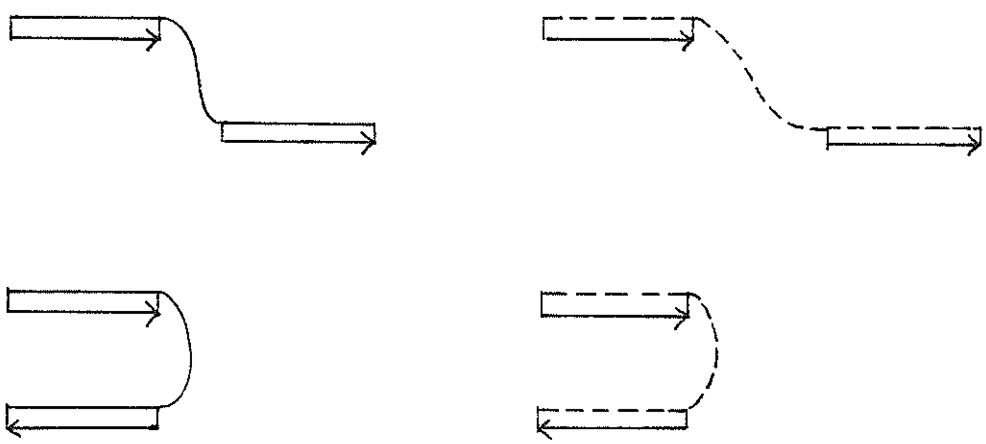


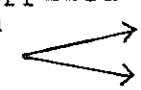
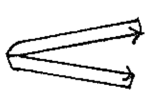
(iv) application of strong and weak braces to non-successive arrow heads and tails:


, etc.

Finally, strong and weak braces can also be applied to arrows that are not linearly sequential[27.01]

(v)



Note. 27.01 Strong and weak braces can be applied also to arrows that do not have a single orientation, as in  to form, say, . We do not consider these in this paper, since they go beyond its intended scope. They are useful in providing alternative graphic structures for representing modifier constructions which are not discussed in this paper, as well as providing graphic representations for possible sentences of languages requiring an extension of the linguistic base obtained by generalizing relative-place ordering q from a linear ordering to a partial ordering.

In order to describe the relationship between the relations that strong and weak braced arrow traces represent and the relations represented by the constituent components of those braced arrow traces, we introduce the following notions:

(a) Let $R_1,\ldots,R_k$ be relations of $n_1,\ldots,n_k$ places on the universe of discourse D, respectively. Then <u>the overlap product of $R_1,\ldots,R_k$</u>, in symbols, $R_1 \otimes \ldots \otimes R_k$, is defined as the relation:

$$\{(a_{11},\ldots,a_{1n_1}, a_{22},\ldots, a_{2n_2}, a_{32},\ldots, a_{3n_3},\ldots, a_{k2},\ldots, a_{kn_k}) \mid (a_{i1},\ldots, a_{in_i}) \in R_i, \text{ for } 1 \leq i \leq n, \text{ and } a_{jn_j} = a_{j+1,1}, \text{ for } 1 \leq j < k\}$$

and the <u>adjacent product of $R_1,\ldots,R_k$</u>, in symbols, $R_1 \dot{X} \ldots \dot{X} R_1$, is defined as the relation:

$$\{(a_{11},\ldots,a_{1n_1}, a_{21},\ldots,a_{2n_2}, a_{31},\ldots,a_{3n_3},\ldots, a_{k1},\ldots,a_{kn_k}) \mid (a_{i1},\ldots,a_{in_i}) \in R_i, \text{ for } 1 \leq i \leq n\}$$

(b) Let R be an m-place relation on the universe of discourse D. Let $1 \leq j_1 < j_2 < \ldots < j_k < m$. Then <u>the restriction of R to the j_1^{st}, j_2^{nd},..., j_k^{th} domains of R</u> is the relation $R_{j_1,\ldots,j_k} = \{(a_{j_1},\ldots,a_{j_k}) \in D^k \mid$ there are $a_1,\ldots,a_{j_1-1}, a_{j_1+1},\ldots, a_{j_2-1}, a_{j_2+1},\ldots, a_{j_k-1}, a_{j_k+1},\ldots, a_m \in D$ such that $(a_1,\ldots,a_m) \in R\}$

(c) Let R be an m-place relation on the universe of discourse D. The complement of R with respect to D is the relation $\overline{R}^D$ such that $\overline{R}^D = \{(a_1,\ldots,a_m) \in D^m \mid (a_1,\ldots,a_m) \notin R\}$

We can then describe the relationship as follows:
Let t be a contiguous sequence and let t_o be a not-necessarily-contiguous sequence of the n arrow traces $t_1,\ldots,t_n$, taken in that order, that represent relations $R_1\ldots,R_n$ of $k_1,\ldots,k_n$ places respectively. Let t' be the result of joining the successive arrow traces $t_1,\ldots,t_n$ of t by an unbarred weak brace, and let t'_o be the result of joining the successive arrow traces $t_1,\ldots,t_n$ of t_o by an unbarred weak brace. Let t'' be the result of joining $t_1,\ldots,t_n$ by an unbarred strong brace.

Let R be the relation represented by t, let R' be the relation represented by t', and let R'' be the relation represented by t'', and let R'_o be the relation represented by t'_o. Let t'^c, t''^c, and t'^c_o be the result of placing a bar on the brace of t', t'', t'_o to form barred braces thereby, respectively, and let R'^c, R''^c, R'^c_o be the relations represented by t'^c, t''^c, t'^c_o, respectively. Then:

(i) R, R', and R'' are each relations of $\sum_{j=1}^{n} k_j - (n-1)$ places, and R'_o is a relation of $\sum_{j=1}^{n} k_j$ places.

(ii) $R'' \subseteq R' = R_1 \otimes \ldots \otimes R_n = R^{27.1} \subseteq (R'_o)_{11} \ldots {}_{1n_1}, {}_{22,\ldots,2n_2,\ldots,k2,\ldots kn_k}$.

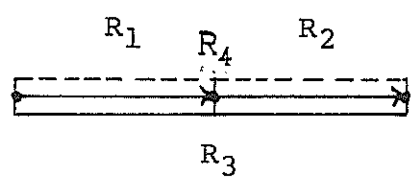
<u>Note 27.1</u>. By rendering R = R' we thereby formalize our decision to interpret unbraced sequences of arrows as weakly braced.

(iii) For every $1 \leq i_1 < \ldots < i_v < \sum_{j=1}^{n} k_j - (n-1)$, if S' (S'') is the relation represented by the subtrace w of $t'(t'')$ such that, for $1 \leq j \leq v$, the j^{th} prong of w overlaps or is overlapped by the $i_j{}^{th}$ prong of t' (t''); and for every $1 \leq i_1 < \ldots < i_v < \sum_{j=1}^{n} k_j$, if S'_o is the relation represented by the subtrace w of t'_o such that, for $1 \leq j \leq v$, the j^{th} prong of w overlaps or is overlapped by the i_j th prong t'_o, then the following hold:

$S' \supseteq R'_{i_1, \ldots, i_v}$,[27.2] and $S'' \supseteq R''_{i_1, \ldots, i_v}$, $S'_o \supseteq R'_{o\, i_1, \ldots, i_j}$, and $S'' \subseteq S$.

(iv) $R'^C = \overline{R'}^D$ and $R''^C = \overline{R''}^D$, and $R'^C = \overline{R'_o}^D$

We can illustrate some of these relationships as they pertain to the set-theoretic meaning of the diagram:



where R_3 is the relation represented by the strongly braced arrow trace shown, R_4 is the relation represented by the weakly braced arrow trace shown, and R_1, R_2 are the relations represented by the constituent arrows common to both the strongly braced arrow trace R_3 and the weakly braced arrow trace R_4 as follows:

(1) $R_3 \subseteq R_1 \otimes R_2 = R_4$

(2) $(R_3)_{1,2} \subseteq R_1$

(3) $(R_3)_{2,3} \subseteq R_2$

That is,

(4) $R_3 \subseteq (R_3)_{1,2} \otimes (R_3)_{2,3} \subseteq R_1 \otimes R_2 = R_4$

We note that (1) cannot be strengthened to read: $R_3 = R_1 \otimes R_2$. For example: given a strongly braced n-term sequence s of arrows (or, equivalently, of two-place strong or weak arrow traces since they are similarly interpreted), and an interpretation of s as, say, an n+1-place relation s*, one can obtain from that interpretation also an interpretation of each of the constituent arrows of s, e.g., the i^{th} arrow s_i of s, as $(s^*)_{i,i+1}$ (that is, the interpretation s_i^* of s_i, as a 2-place relation, is the 2-place relation $(s^*)_{i,i+1}$). However, one cannot obtain an interpretation s^* of the strongly braced n-term sequence s of arrows from the interpretations of its constituent arrows $s_{1,2}, \ldots, s_{n-1,n}$, namely, the interpretations $(s_{1,2})^*, \ldots, (s_{n-1,n})^*$, as the overlap product $(s_{1,2})^* \otimes \cdots \otimes (s_{n-1,n})^*$; rather one can only obtain an inclusion relationship $s^* \subseteq (s_{1,2})^* \otimes \cdots \otimes (s_{n-1,n})^*$. This overlap product is, rather, the proper interpretation of the corresponding weakly braced arrow trace.

Note 27.2. We require $\supseteq$ rather than = to allow the possibility that, for example, John could love without his loving anything, or that John could love Mary without his loving her at any particular place, at any particular time, for any particular purpose, and so on.

2.3. Dotted Lines and Their Interpretation. A dotted line (................) represents the identity relation, and a barred dotted line (------+-------) represents the difference relation. That is, a dotted line joining two graphic components represents that those components represent the same entity, that is, the same element or relation, and a barred dotted line joining them represents that those components represent different entities. In particular, a dotted line joining two points represents that those points represent identical elements of the universe of discourse D, and a dotted line joining two (strongly or weakly braced) arrow traces represents that those arrow traces represent identical relations among elements of D. In this subsection we will introduce several uses of dotted lines with this interpretation.

Diagrammatic representation of the copula. The copula, in English, is usually signalled by a form of the verb "to be", as occurring in either the identity usage (Socrates is the teacher of Plato), or in the predicative usage (Socrates is a man). In either usage, the copula is interpreted as the identity relation on elements of D, hence is diagrammatically represented as a dotted line.

The copula has a special status insofar as it is a logical rather than a lexical relation. (There are further logical relations[28] as well but they can be represented without introducing further special sorts of symbols, being representable

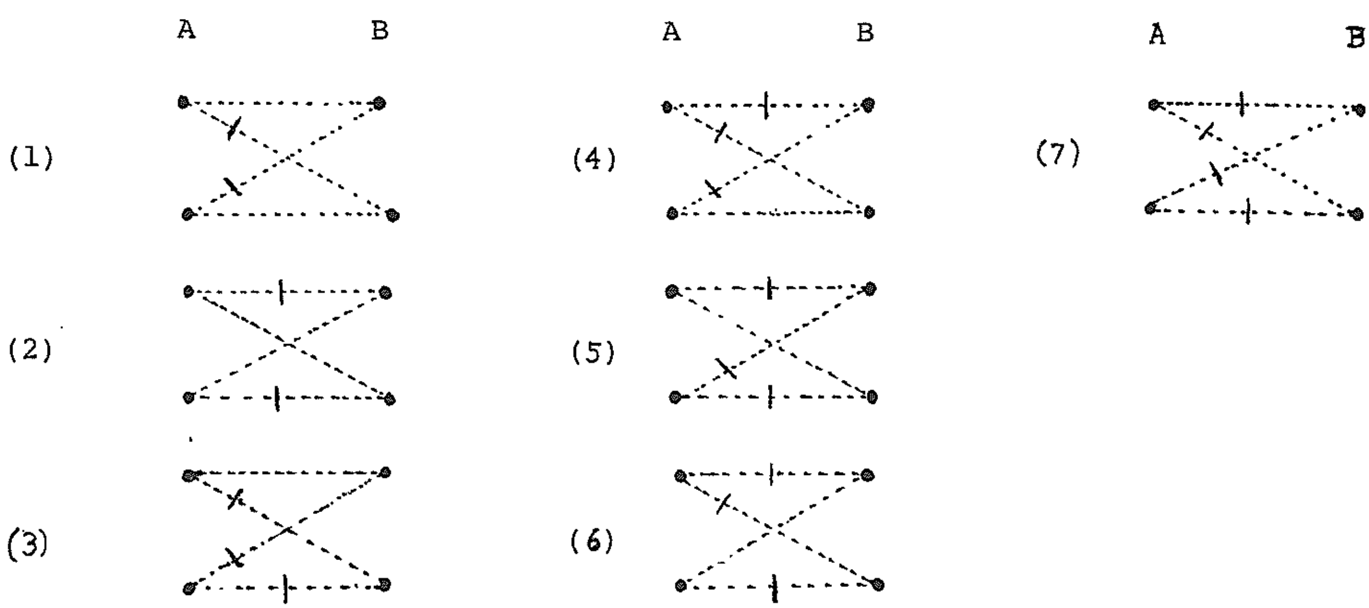
Note 28. For example, comparative relations, expressed in English by phrases like "is taller than", which signals a logical relation built onto a lexical base "tall".

by special configurations of the points, arrows, and dotted lines already introduced).

Let us briefly illustrate the use of dotted lines to represent the copula by means of a brief example which follows the pattern of illustration used in Section 1.

Assume that the universe of discourse D consists of all living things and that the elements a_1, a_2 comprise all the men in D, and that the elements b_1, b_2 comprise all the mortals in D. Let us assume further that the relation R is the relation of <u>being identical to</u>. Then A is the point bank representing all men, and B is the point bank representing all mortals.

Let $\mathcal{L}$ then be the set of all possible event particular diagrams constructible on the pair of point banks A, B as described:



The sentence:

(i) All men are mortal

corresponds to an ED whose constituents are the elements of the set {(1),(2)}

The sentence:

(ii) Some men are mortal

corresponds to an ED whose constituents are the elements of the set {(1),(2),(3),(4),(5),(6)}

The sentence

(iii) No men are mortal

corresponds to an ED whose constituents are the elements of the set {(7)}

The sentence

(iv) Exactly one man is a mortal

corresponds to an ED whose constituents are the elements of the set {(3),(4),(5),(6)}

The sentence

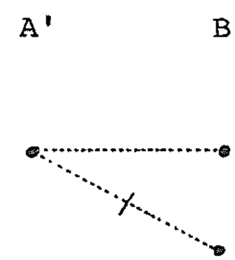
(v) At most one man is mortal

corresponds to an ED whose constituents are the elements of the set {(3),(4),(5),(6),(7)}

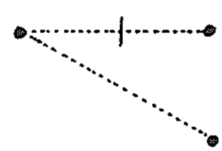
As a second sort of example involving dotted lines, we continue to regard the universe of discourse D as consisting of all living things, and the point bank B as representing the set of all mortals in D, but we now assume that the point bank A' represents the (singleton) set consisting of the single individual element Socrates, designated here as "a":

Let now $\mathcal{L}$' be the set of all possible event particular diagrams constructible on the pair of point banks A', B

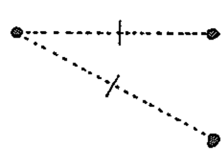
(8)



(9)



(10)



The sentence

(i) Socrates is mortal

corresponds to an ED whose constituents are the elements of the set: { (8), (9) }

The sentence

(ii) Socrates is not mortal

corresponds to an ED whose constituents are the elements of the set: {(10)}

We note here an important distinguishing characteristic of dotted lines as compared to arrows in the construction of event particular diagrams: at most one unbarred dotted line can issue from a given point, and at most one unbarred dotted line can terminate at a given point. This reflects the intended interpretation[29] of the dotted arrow as representing the identity relation, which is a one-to-one relation. Thus at most one unbarred dotted line can join two given points.

In the above use of dotted lines between points, dotted lines joined points in different point banks of the same EPD, to represent that those points represented the same element of D.

A second use of dotted lines between points is to diagrammatically represent that two points in different EPDs represent the same element of D. This second use is essential for the proper representation of entailment, and will be described later in this section.

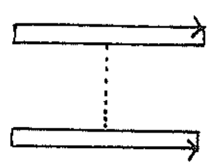
A dot path is a dotted line together with the points or arrow traces it joins.

Note 29. For a general discussion of interpretations of RBSN components, see Section 2.5.

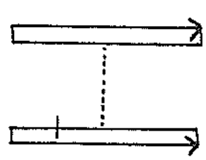
2.4 Graphic Relationships Among Arrow Paths and their Set-Theoretic Interpretation.

Just as a dotted line joining two points signifies that those points represent identical elements of D, a dotted line joining two arrow traces signifies that those arrow traces represent the same relation. Since a barred arrow trace represents the complement of the relation represented by the unbarred version of that arrow trace, a dotted line joining two arrow traces, one of which is barred and the other of which is unbarred signifies that those arrow traces represent complementary relations with respect to the domain of discourse D, that is, they represent relations that are complements of each other with respect to D in the following sense:

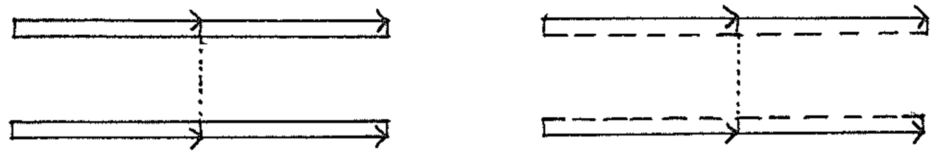
For example,

(i) 

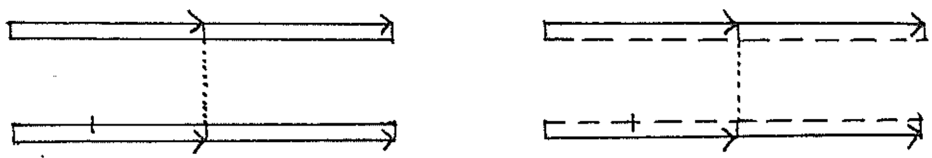
signifies that the upper and lower arrow traces represent the same two-place relation, whereas

(ii) 

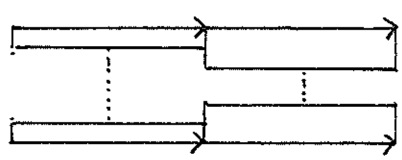
signifies that the upper and lower arrow traces represent complementary two-place relations, that is, the upper relation holds among given elements if and only if the lower relation fails to hold among them.

(iii) 

signifies that the upper and lower arrow traces represent the same three-place relation, whereas

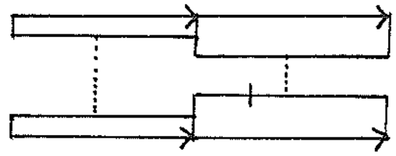
(iv) 

signifies that the upper and lower arrow traces represent complementary three-place relations.

(v) 

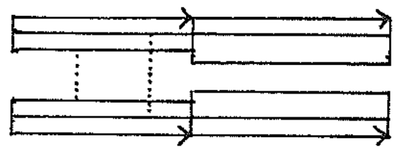
signifies that each pair of upper and lower arrow traces joined by dotted lines represents the same two-place relation

(vi)



signifies that the first (i.e., left) pair of arrow traces joined by dotted lines represents the same two-place relation, and that the second (i.e., right) pair of arrow traces joined by dotted lines represents complementary two-place relations.

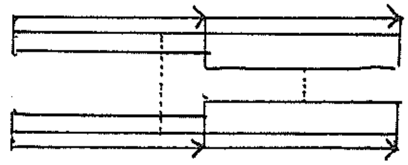
(vii)



signifies that the first (i.e., left) pair of arrow traces joined by dotted lines represent the same two-place relation, and that the upper arrow trace and the lower arrow trace represent the same three-place relation.

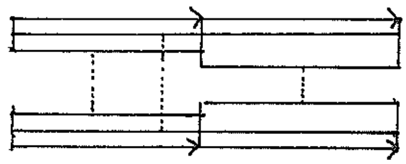
We note that the above diagram (vii) does not signify that the second, (i.e., right) pair of arrow traces also represent the same relation, for they may not. Precisely analogous remarks apply to:

(viii)



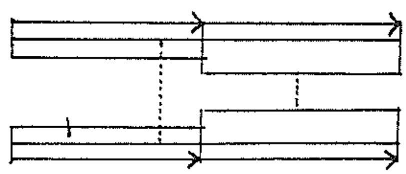
On the other hand,

(ix)



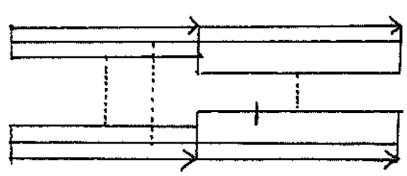
signifies that each pair of arrow traces joined by dotted lines represent the same relation so that (ix) carries more meaning than either (vii) or (viii). We note in passing that

(x)



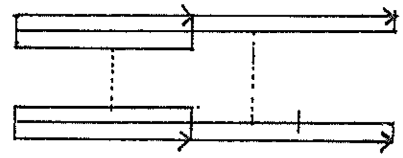
and

(xi)



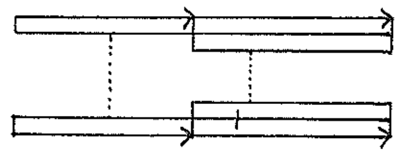
are impossible, whereas

(xii)



and

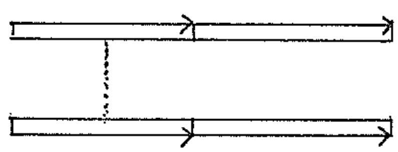
(xiii)



are possible.

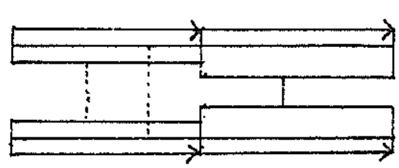
This means then that if it is the case that

(xiv)



then it is also the case that

(xv)



That is to say, if a dotted line joins two strong arrow traces p_1 and p_2, then it is implicit that a dotted line joins each corresponding pair of sub-arrow traces of p_1 and p_2.

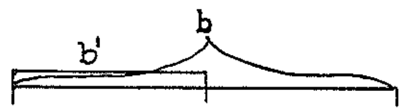
2.5 Graphic Structure of EPDs and EDs as Arrays, and Their Set-Theoretic Interpretation. By an array I mean a graphic configuration the meaningful parts of which are called the constituents of the array. An array, as such, is to be distinguished from the set of its constituents. A given array has a graphic structure in the same sense that a given English letter, word, or sentence has a graphic structure.[29.1] This structure may or may not be specified. We impose a particular "standard form" structure on EPDs and EDs for definiteness.

By a linked array I mean an array K with arrow paths, EPDs, or EDs as constituents, and having dotted lines joining, i.e., "linking", various points and arrow traces occurring within them. The significance of a linked array is that points that are explicitly joined by a dotted line are interpreted as representing identical elements of the underlying domain of discourse D, and arrow traces that are explicitly joined by a dotted line are interpreted as representing identical relations among elements of the underlying domain of discourse D.

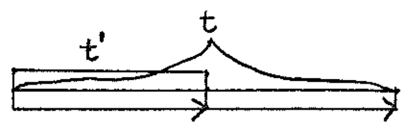
A brace b' is a sub-brace of a brace b if and only if b and b' are either both strong or both weak and all prongs of b' overlap or are overlapped by prongs of b.

Note 29.1 See Note 9.1.

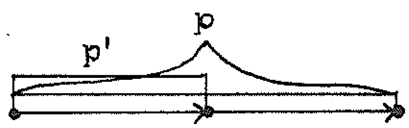
For example, b' is a sub-brace of b in



t' is a sub-trace of t in



and p' is a sub-path of p in



An arrow trace t' is a <u>sub-arrow trace</u> of an arrow trace t if and only if the brace of t' is a sub-brace of the brace of t, and the arrow sequence underlying t' is a subsequence of the arrow sequence underlying t, and t' is a <u>proper</u> sub-arrow trace of t if, in addition, $t' \neq t$.

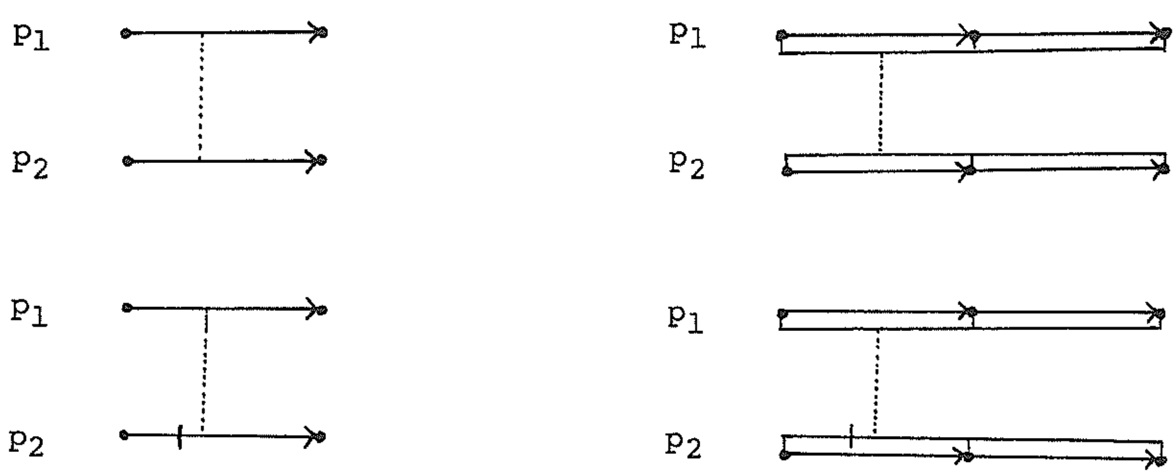
An arrow trace t' is <u>analogous to</u> an arrow trace t if and only if t' is like t except that the brace of t is strong (weak) and the brace of t' is weak (strong).

An arrow path p' is a <u>sub-arrow path</u> of an arrow path p if and only if the arrow trace of p' is a sub-arrow trace of the arrow trace of p and p' is a <u>proper</u> sub-arrow path of p if, in addition, $p' \neq p$.

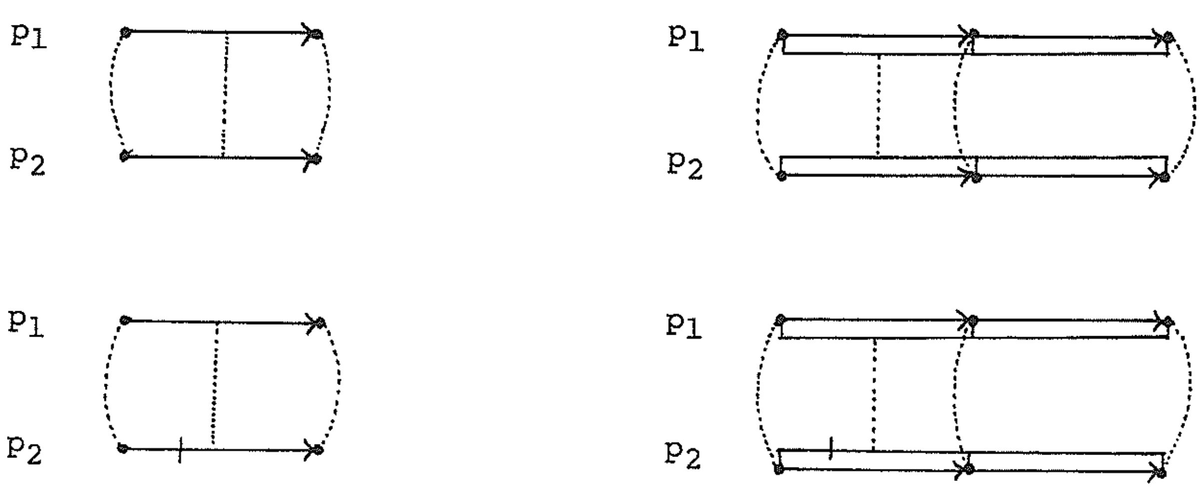
A <u>major sub-arrow-path (sub-arrow-trace)</u> p' of an arrow path (trace) p is a sub-arrow path (trace) p' of p such that there is no proper sub-arrow path (trace) p'' of p of which p' is a proper sub-arrow-path (sub-arrow-trace).

Two arrow paths p_1 and p_2 are <u>similarity linked</u> if and only if (i) they have the same length m, and (ii) the arrow trace of p_1 is linked by a dotted line to the arrow trace of p_2; and are <u>strictly similarity linked</u> if, in addition, (iii) for each $1 \leq i \leq m$, the i^{th} point of p_1 is linked by a dotted line to the i^{th} point of p_2, and are <u>exactly similarity linked</u> if, in addition, (iv) both are barred or both are unbarred (i.e., if both p_1 and p_2 have the same "signing").

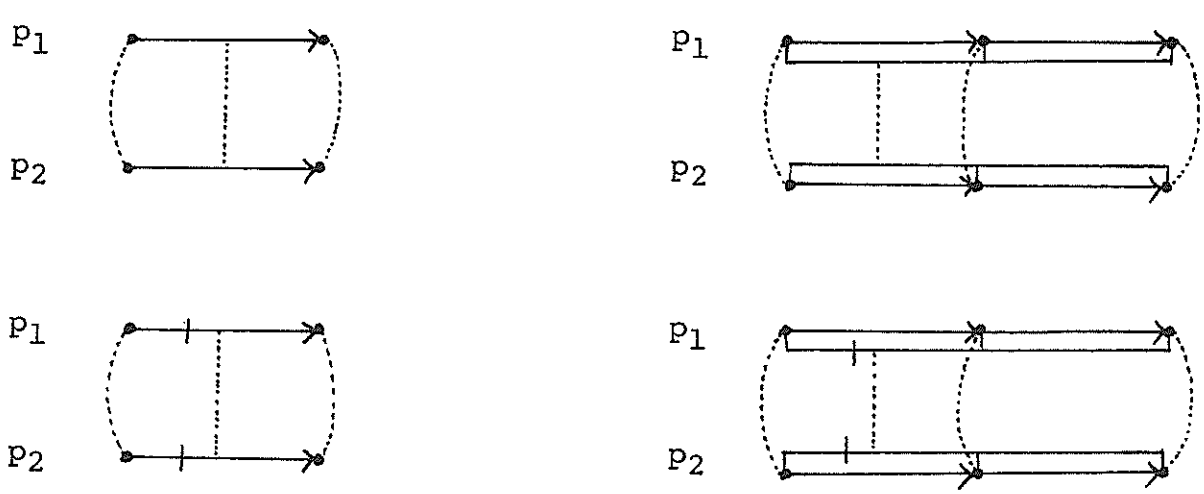
For example, p_1, p_2 are <u>similarity linked</u> in each of



and are <u>strictly similarity linked</u> in each of



and are <u>exactly similarity linked</u> in each of



An <u>event particular diagram</u> (EPD) A is an array of similarity linked arrow paths of length m, for some positive integer m, such that (i) for all m-tuples $(a_1,\ldots,a_m)$ such that for all $1 \leq i \leq m$, a_i is a point of some arrow path p_i of A, there is exactly one arrow path p of A whose point trace is $(a_1,\ldots,a_m)$. For each $1 \leq i \leq m$, the <u>i^{th} point bank of A</u> is an array of the i^{th} points of the point traces of the arrow paths of A.

Two event particular diagrams (EPDS) E_1, E_2 are <u>similarity linked</u> if and only if there is a one-one onto correspondence between the arrow paths of E_1 and the arrow paths of E_2 such that corresponding arrow paths are strictly similarity linked, but not all corresponding arrow paths are exactly similarity linked.

An <u>event diagram</u> (ED) is an array of similarity linked event particular diagrams.

We note that any two distinct event particular diagrams E_1, E_2 in an event diagram E are "inconsistent" in the sense that there is an arrow path p in E_1 and an arrow path q in E_2 such that p and q have the same point trace and there is a dotted line joining some major sub-arrow paths of p and q which are barred in one of p,q and unbarred in the other.

An event particular diagram E is in standard form if the points of each point bank of E appear as a column, and the point banks of E are horizontally arranged among themselves.

An event diagram is in standard form if each of its constituent event particular diagrams is in standard form and it appears as a two-dimensional array of horizontally arranged columns of event particular diagrams.

An RBSN is an array of event diagrams together with dotted lines joining points and arrow traces of the constituent event diagrams of the RBSN.

An RBSN is in standard form if its constituent EDs are in standard form.

We are now concerned to formulate set-theoretic interpretations of EDs in terms of which we can state an intuitive entailment paradigm that is to be subsequently reformulated wholly in terms of the graphic patterns within an RBSN that contains the entailing and entailed EDs as constituents.

In the foregoing discussions the relationships between network components and the set-theoretic entities that they represented were implicit in our diagrams and in our description of their meanings.

In order to state the intuitive entailment paradigm, that is subsequently to be explicated in purely graphic terms, we need to make explicit reference to a representation function that assigns set-theoretic entities to network components. We define such a function recursively as follows:

Let α be an RBSN and let D be a universe of discourse. A representation function on α to D is a function * such that:

(i) if y is a point of α , then $y^* \in D$

(ii) if y is an arrow trace of α of length m, then $y^* \subseteq D^m$; if y', y'' are analogous arrow traces of x such that y' is strong and y'' is weak and if $y_1,\ldots,y_n$ are the major sub-arrow traces of y', then $(y')^* \subseteq (y'')^* = y_1^* \otimes \cdots \otimes y_n^*$; if y is an arrow trace of length m, and if y^o is a sub-arrow trace of y such that, if $p_{i_1},\ldots,p_{i_v}$ are all the prongs of y that overlap the prongs of y^o, where $i_1<\ldots<i_v$, and which are such that, for each $1 \leq j \leq v$, the j^{th} prong of yo overlaps the $i_j{}^{th}$ prong of y, then $y^{o*} = (y^*)_{i_1 \cdots i_v}$.

(iii) if y is a dotted line of α , then y* is the identity relation on D and on D^m, for every positive integer m.

(iv) if y' is a barred arrow-trace or a barred dotted line of α and y is obtained from y' by deleting the bar, then $(y')^* = \overline{y^*}^D$.

(v) if y is an m-place arrow path of α with arrow trace x and point trace $(x_1,\ldots,x_m)$, then y holds under * if and only if $(x_1^*,\ldots,x_m^*) \in x^*$.

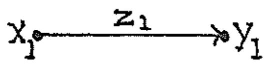
(vi) if y is an unbarred or barred dot path composed of an unbarred or barred dotted line x of α joining points or arrow traces x_1, x_2 of α, then y <u>holds under *</u> if and only if $x_1^* = x_2^*$ or $x_1^* \neq x_2^*$, according as, respectively, x is unbarred or barred.

(vii) An event particular diagram x occurring within an event diagram y of α holds under * if and only if each of the constituent arrow paths or dot paths in some reduction of x relative to y holds under * and each of its linking dot paths holds under *.

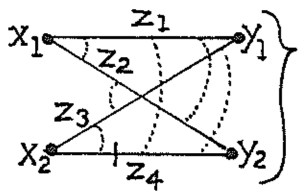
(viii) An event diagram of α holds under * if and only if at least one (equivalently, exactly one) of its constituent EPDs holds under * and each of its linking dot paths holds under *.

(ix) A linked array of event diagrams of α holds under * if and only if each of its constituent event diagrams holds under * and each of its linking dot paths holds under *.

The following consequences of the above definition can be easily seen to follow:

(1)  holds under * if and only if (1'):

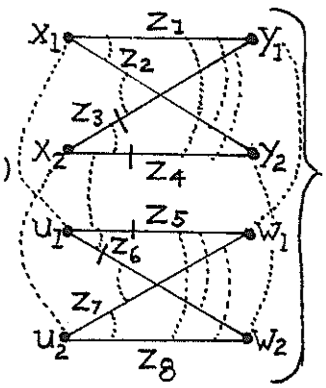
$(x_1^*, y_1^*) \in z_1^*$

(2)  holds under * if and only if (2'):

$(x_1^*, y_1^*) \in z_1^*$,

$(x_1^*, y_2^*) \in z_2^*$, $(x_2^*, y_1^*) \in z_3^*$, $(x_2^*, y_2^*) \in z_4^*$,

$z_1^* = z_2$, and $z_3^* = z_4^* = \overline{z_1^*}^D$.

(3)  holds under * if and only if (3'):

$(x_1^*, y_1^*) \in z_1^*$,

$(x_1^*, y_2^*) \in z_2^*$, $(x_2^*, y_1^*) \in z_3^*$, and

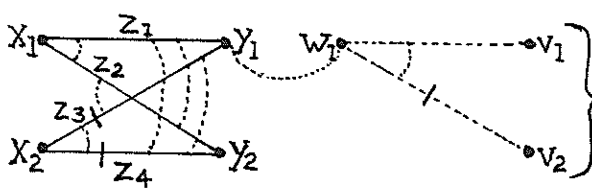
$(x_2^*, y_2^*) \in z_4^*$; or $(u_1^*, w_1^*) \in z_5^*$,

$(u_1^*, w_2^*) \in z_6^*$, $(u_2^*, w_1^*) \in z_7^*$, and

$(u_2^*, w_2^*) \in z_8^*$, and $x_1^* = u_1^*$, $x_2^* = u_2^*$,

$y_1^* = w_1^*$, $y_2^* = w_2^*$, $z_1^* = z_2^* = z_7^* = z_8$, and

$z_3^* = z_4^* = z_5^* = z_6^* = \overline{z_1^*}^D$

(4)  holds under * if and only if (4'):

$(x_1^*, y_1^*) \in z_1^*$, $(x_1^*, y_2^*) \in z_2^*$,

$(x_2^*, y_1^*) \in z_3^*$, $(x_2^*, y_2^*) \in z_4^*$,

$w_1^* = v_1^*$ $w_1^* \neq v_2^*$, and

$z_1^* = z_2^*$, $z_3^* = \overline{z_4^*}^D = z_1^*$, $y_1^* = w_1^*$ [30]

Note 30. The labels "x_1, "y_1, etc., occurring in these diagrams are here to be considered as names of the points and arrow traces to which they are affixed, rather than names of elements of D or relations on D as was the case in Section 1.

We can now state the intended entailment paradigm as follows:

Let $ED_1, \ldots, ED_m$, ED_{m+1} be (interlinked) event diagrams within an RBSN α : Then $ED_1, \ldots, ED_m$ entail ED_{m+1} (relative to α) if and only if for every universe of discourse D, and for every representation function * on α to D, if $ED_1, \ldots, ED_m$ hold under *, then ED_{m+1} also holds under *.

Our concern is to formulate this paradigm wholly in terms of the diagrammatic patterns holding among $ED_1, \ldots, ED_m$ and ED_{m+1}; that is, our concern is to give an explicit diagrammatic characterization of entailment, permitting thereby a purely diagrammatic means of determining instances of (valid) entailment. We describe the intended characterization in the next subsection. We introduce the essential notions required for this characterization.

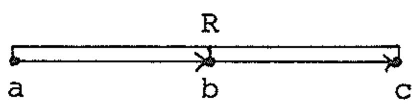
An m-place arrow path p' is an explicit resultant of an n-place arrow path p if and only if p' and p are both unbarred (barred), $n \geqslant m$ ($n \leqslant m$), it is not the case that p (p') is weak and p' (p) is strong, the point trace of p' (p) is a sub-tuple of the point trace of p (p'), and the arrow trace of p' (p) is a sub-arrow trace of the arrow trace of p(p').[31]

Note 31. We remark here on the asymmetry between the barred and unbarred cases in the definition of explicit resultant.

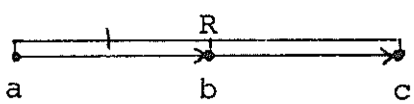
This asymmetry derives from a general situation which can be seen in the following examples:

(i) John gave the book to Mary
(ii) John did not give the book to Mary
(iii) John gave the book
(iv) John did not give the book
(v) The book was given to Mary
(vi) The book was not given to Mary

Under the dominant reading of each of (i)-(vi), (i) entails each of (iii) and (v), but is entailed by neither (iii) nor (v); indeed, (i) is not entailed by the conjunction of (iii) and (v). On the other hand, (ii) entails neither (iv) nor (vi) but (ii) is entailed by each of (iv) and (vi). The general situation regarding the semantic structure of sentences which underlie the patterns of inter-entailments noted here among sentences (i)-(vi) is reflected in the following interconnections among the set-theoretic interpretations of arrow paths:

(1) 

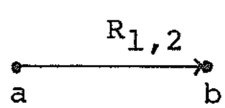
is interpreted as meaning that $(a,b,c) \in R$, that $(a,b) \in R_{1,2}$, and that $(b,c) \in R_{2,3}$.

(2) 

is interpreted as meaning that $(a,b,c) \in \overline{R}^D$, that $(a,b) \in (\overline{R}^D)_{1,2}$, and that $(b,c) \in (\overline{R}^D)_{2,3}$.

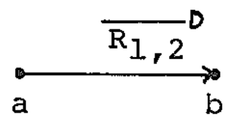
While $(a,b) \in R_{1,2}$ can be regarded as meaning that a,b, taken in that order, stand in the relation $R_{1,2}$, $(a,b) \in (\overline{R}^D)_{1,2}$ can similarly be regarded as meaning that a,b, taken in that order, stand in the relation $(\overline{R}^D)_{1,2}$, but not as meaning that a,b, taken in that order, fail to stand in the relation $R_{1,2}$, because generally, $\overline{R_{1,2}}^D \neq (\overline{R}^D)_{1,2}$.

Accordingly, while we regard (1) as "entailing"

(1') 

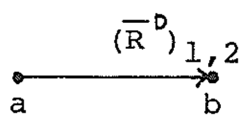
,

we do not analogously regard (2) as "entailing"

(2') 

,

but only as "entailing"

(2'') 

.

For example, (1) could correspond to the English sentence "John gave the book to Mary", (1') to "John gave the book", (2) to "John did not give the book to Mary", (2') to "John did not give the book" in the sense that John did not give the book to anyone, and (2'') to "John did not give the book" in the sense that John did not give the book to some particular person, i.e. that there is some person such that John did not give the book to that person.

The dominant reading, of course, of "John did not give the book" is that reading which is equivalent to the dominant reading of "John did not give the book to anyone", and is not equivalent to any reasonable reading of "John did not give the book to some particular person". There appears to be no sentence in English that refers only to <u>John</u>, <u>the book</u>, and <u>giving</u>, where <u>giving</u> is syntactically rendered as a two-place relation, whose dominant reading <u>is</u> equivalent to the dominant reading of "John did not give the book to any particular person", in the same way that "John did not give the book" has a dominant reading equivalent to that of "John did not give the book to anyone".

The meanings of the labels on the diagrams exhibited in this footnote revert back to our usual usage in this paper, whereby the labels affixed to points and arrow traces in diagrams designate, not those points or arrow traces, but rather designate those elements and relations on the domain of discourse that those points and arrow traces represent.

An RBSN α is complete with respect to explicit resultants if and only if for every arrow path p that occurs in α , all explicit resultants of p also occur in α .

An m-place arrow path p'' of an RBSN α is an implicit resultant of an m-place arrow path p of α relative to α if and only if p'' is exactly similarity linked to some explicit resultant p' of p, where all dotted line links are in α.

Finally, an arrow path p' of α is a resultant relative to α of an arrow path p of α if and only if p' is an explicit resultant of p or an implicit resultant of p, relative to α .

The rationale of our definition of sub-arrow path can now be stated:

Let p,p' be arrow paths of an RBSN α . Then the following are equivalent:

(i) p' is a resultant of p

(ii) For any universe of discourse D, and for any representation function * from α to D, if p holds under *, then p' also holds under *.

In order to treat entailment involving more than one premise, we need to allow arrow paths to be resultants of a more "generalized" arrow path .

A generalized arrow path p^ of an RBSN α on EPDs $E_1,\ldots,E_n$ of α is a sequence of alternating arrow paths and dotted lines of α joining point $p_1,\ldots,p_v$ of α such that for each $1 \leq i \leq v$, p_i is a resultant of some arrow path of some E_j, $1 \leq j \leq n$, the last point of p_i is joined to the first point of p_{i+1} by a dotted line, and all arrows of p^ are similarly oriented (i.e., point in the same direction).

Let * be a representation function on an RBSN α to a universe of discourse D.

We extend the definition of "holds under *" to apply to generalized arrow paths of α: A generalized arrow path p^ of α holds under * if and only if each of its constituent arrow paths and dot paths holds under *.

A resultant of a generalized arrow path p^ is an arrow path p that is either a sub-arrow path of some arrow path of p^ or else is a resultant of an arrow path obtained from some subsequence p' of p^ by replacing every configuration of p^ consisting of a dotted line and the two points they join by one or the other of these points.

The rationale of our definition of resultant is based on the following observation:

Let p^ be a generalized arrow path of an RBSN α and let p' be an arrow path of α. Then the following are equivalent:

(i) p' is a resultant of p^
(ii) For any universe of discourse D, and for any representation function * from α to D, if p^ holds under *, then p' holds under * as well.

Let $EPD_1,\ldots,EPD_k$ be similar EPDs and let $p_1,\ldots,p_j$ be arrow paths such that $p_1,\ldots,p_j$ are each constituents of each of $EPD_1,\ldots,EPD_k$. Then $\{p_1,\ldots,p_j\}$ is a reduction set for $EPD_1,\ldots,EPD_k$ if and only if all possible combinations of signings of the constituent arrow paths of $EPD_1,\ldots,EPD_k$ not similar to any of $p_1,\ldots,p_j$ occur among $EPD_1,\ldots,EPD_k$, where the "signings" of a given arrow path are the unbarred and barred variants of that arrow path.

We need to introduce another simplification of EDs, as follows:

If x is a constituent EPD of an ED y, and if $\pi = \{\pi_1, \pi_2\} = \{\{p_1, \ldots, p_n\}, \{p_{n+1}, \ldots, p_{n+m}\}\}$ is a partition of the constituent arrow paths $p_1, \ldots, p_n, p_{n+1}, \ldots, p_{n+m}$ of x, then the <u>internal reduction of y determined by x and π</u> is that ED y' obtained from y by replacing that subarray y^o of y whose constituents are all and only those EPDs x'' of y such that some signing variant

$p_1, \ldots, p_n$ by that EPD whose constituents are precisely $p_1, \ldots, p_n$ (in the sense of being exactly similarity linked to the arrow paths $p_1, \ldots p_n$ of x).

The <u>internal reduction IR(y) of y</u> is that ED obtained from y by obtaining the internal reduction of y determined by some EPD x of y and some partition π of the arrow paths of x, and repeating this process till no further proper internal reductions are possible. A unique ED y* is obtained in this way relative to y and we designated it as <u>the internal reduction IR(y) of y</u>.

$p'_{n+1}, \ldots, p'_{n+m}$ of $p_{n+1}, \ldots p_{n+m}$ occurs in x'' and such that every arrow path p of x'' distinct from $p'_{n+1}, \ldots p'_{n+m}$ occurs among

2.6 Simple EDs and the Basic Entailment Form

We will refer to EPDs and EDs of the type we have thus far described as simple, and distinguish them from a further type of EPD and ED (to be introduced subsequently) which we will refer to as complex.

Accordingly we will first formulate a basic entailment form that defines the conditions under which simple EDs entail a (given) simple ED, and will later (on page 112) generalize this to an extended entailment form that defines the conditions under which simple or complex EDs entail a given simple or complex ED.

The Basic Entailment Form. Let $ED_1,...,ED_m$, ED_{m+1} be EDs of an RBSN α. Then $ED_1,...,ED_m$ entail ED_{m+1} relative to α if and only if, for every consistent choice of constituent EPDs $E_1,...,E_m$ of $ED_1,...,ED_m$ respectively, there is a constituent EPD E_{m+1} of ED_{m+1} such that each of the arrow paths in some reduction set E'_{m+1} of E_{m+1} relative to ED_{m+1} is a resultant of a generalized arrow path of $E_1,...,E_m$.[32]

Note 32. This entailment form and its extensions can also be generalized to comprehend a form of "probabilistic entailment" as follows: let t be the number of choices of EPDs among $ED_1,...,ED_m$, i.e., t is the cardinality of the Cartesian product $ED_1 x...xED_m$; and let $r \leq t$ be the number of elements $(EPD_1,...,EPD_m) \in ED_1 x...xED_m$ such that EPD_{m+1} is a resultant of $EPD_1,...,EPD_m$. Under these conditions we say that $ED_1,...,ED_m$ r/t-entails ED_{m+1}. The rationale for this type of probabilistic entailment is roughly as follows:

Each ED depicts the set of all circumstances depicted by the constituent EPDs of that ED under which it can be true, subject only to the cardinality assumptions carried by the index function h. If for each element $(EPD_1,...,EPD_m) \in ED_1 x...xED_m$, $EPD_1,...,EPD_m$ collectively depict a circumstance which is depicted by some EPD $\in ED_{m+1}$, then $ED_1,...,ED_m$ entail ED_{m+1}. If there are t total elements in $ED_1 x...xED_m$ and r of them collectively depict some circumstance depicted by some EPD $\in ED_{m+1}$, then $ED_1,...,ED_m$ r/t-entail ED_{m+1}, which is to say that if $ED_1,...,ED_m$, ED_{m+1} represent, respectively, the sentences $e_1,...,e_m$, e_{m+1}, then the probability is r/t that e_{m+1} is true

In the special case of our earlier examples in Section 1 , $m=1$, and $ED_1=ED_m \subseteq ED_{m+1}=ED_2$. In this special case, every EPD of ED_1 is an EPD of ED_2; thus, since the relation of "being a resultant of" is reflexive, we have that ED_1 entails ED_2. Thus, if an ED X is a subset of an ED Y, then X entails Y; that is, if X is set-theoretically contained in Y, then X is implicitly contained in Y.

2.7 Examples of Simple Event Diagrams In the following examples, we indicate (but do not exhibit) the intended syntactic representation of a given natural language sentence e by superscripting a prime symbol "'" to that sentence, obtaining thereby "e'". For simplicity in exposition we have chosen sample sentences whose dominant reading permits the association of point banks $A_1, A_2, \ldots, A_m$ with the successive noun phrases they represent, that is, in accord with the left-to-right order of occurrence of those noun phrases in e.[33,33.1] Accordingly, we do not

under the assumption that each of $e_1, \ldots, e_m$ is true (and under the cardinality assumptions carried by the index function h). Furthermore, we can also employ sampling methods for estimation of the proportion $^r/_t$ in the population $ED_1 x \ldots x ED_m$ from a random sample relative to given confidence levels, which is useful when the number of elements in $ED_1 x \ldots x ED_m$ is very large.

Note 33. See Appendix for the orientation of this special case to the more general case.

Note 33.1 More generally, we have included here only examples involving EDs of relatively simple structure and with relatively simple interconnections among them. The techniques of ATR provide for the construction of EDs of much more general kinds than are described or illustrated here which would be required to represent entailment among natural language sentences of more varied sorts.

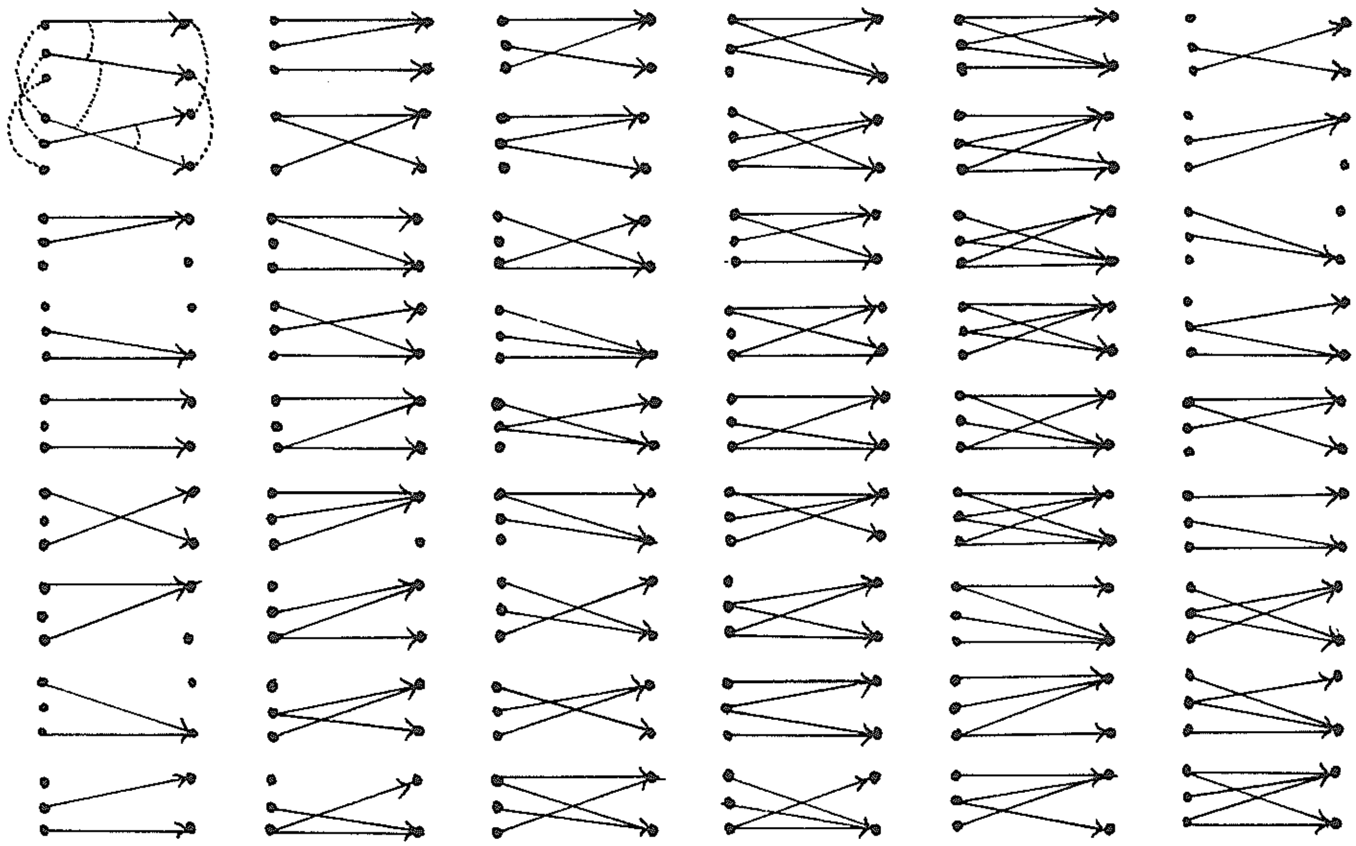
need to label the point banks in the diagrams insofar as the labels can be directly identified by virtue of this convention. The number of points in a point bank corresponding to a given noun phrase is specified in practice in terms of a function, called an index function, that assigns to every noun phrase a positive integer. Accordingly, in the following examples, the intended index function on noun phrases is indicated by the number of points in the respective point banks representing those noun-phrases.[33.2]

Note 33.2 In order to keep the sample EDs readily examinable, we have used small, i.e., restrictive, index functions. In practice, somewhat larger index functions would be required, which would be offset by a variety of simplifications that could be employed to markedly reduce EDs and yet carry the same information. For example, a fairly substantial sort of simplification would be obtained by taking, instead of a full event diagram, only an array of "representatives" of its constituent event particular diagrams, as determined by the equivalence relation "is a rearrangement of", described in Section 1 earlier. For most typical cases, this will reduce the event diagram to a small fraction of its original size. For a further discussion of index functions, see the Appendix.

The event diagram[34,34.1] of:

(a) At least two men love at least one woman

corresponding to its dominant reading then is:



Note 34: In order to follow the examples, it is sufficient that the reader "read" the sentences occurring in the examples in an intuitive sense, and match those readings against an intuitive appraisal of what the diagrams represent. This appeal to intuition is supplanted in ATR by an explicit procedure for converting readings (e',s) of natural language sentences e into corresponding event diagrams for any given finite index function h.

Note 34.1: Our practice in displaying event diagrams will be (i) to display them in standard form, (ii) to display unbarred arrow paths and dot paths only, (iii) to indicate the dotted line linkages among EPDs only among the first several EPDs displayed and, within a given EPD, to indicate only those dotted line linkages among arrow paths from which the remaining dotted line paths can be constructed by transitivity, and (iv) to exhibit all the EPDs of the ED if the number of constituent EPDs is not very large; otherwise we exhibit only a sample subset of those EPDs.

Examples of Further Event Diagrams of Sample Sentences

Consider:

(1) At least one man loves Mary

(2) At least two men love Mary

(3) At most one man loves Mary

(4) At most two men love Mary

(5) At least one man loves

(6) At least two men love

(7) At most one man loves

(8) At most two men love.

Now, under the dominant normal (by my ear) readings of (1)-(8), we have:

(2) entails (1)

(1) entails (5)

(2) entails (6)

(3) entails (4)

(7) entails (3)

(8) entails (4)

Consider further

(9) Exactly one man loves Mary

(10) Exactly two men love Mary

Then, under the further dominant normal readings of (9) and (10) we have:

(1) and (3) together entail (9)

(2) and (4) together entail (10)

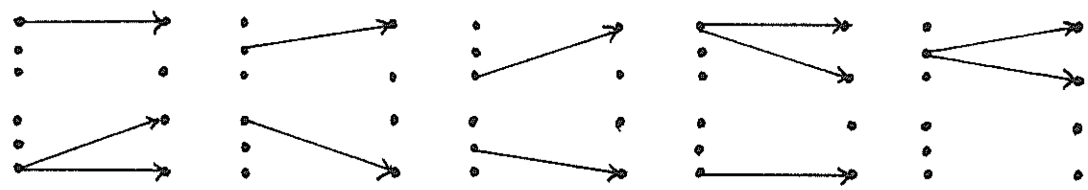
The event diagram corresponding to the <u>negation</u> of:

(a) At least two men love at least one woman

under its dominant reading, that is, that reading which is equivalent to the dominant reading of

(b) At most one man loves at least one woman

is the complement of the above ED with respect to the set of EPDs similar to the EPDs of that earlier ED:[35]



Add now:

(11) Exactly one man loves

(12) Exactly two men love

Then, under the further dominant normal readings of (11) and (12) we have:

(5) and (7) together entail (11)

(6) and (8) together entail (12)

Consider further:

(13) Mary is a waitress

(14) John gave at most two books to Mary

(15) John gave at most two books to a waitress

<u>Note 35</u>. We note that the number of possible EPDs with point banks of 3 and 2 elements respectively is $2^{3\times 2} = 2^6 = 64$, and that this is just the sum of the EDs corresponding to sentences (a) and (b) above: namely 64 = 54 + 10 EPDs.

(16) John gave at most one book to a waitress

(17) John gave at most one book

(18) John did not give any books

(19) John gave at least two books to Mary

(20) John gave at least two books to a waitress

(21) John gave at least one book to a waitress

(22) John gave no books to Mary

(23) All men are mortal

(24) Socrates is a man

(25) Socrates is mortal

(26) Socrates is a philosopher

(27) A philosopher is mortal

(28) Socrates knows a philosopher

(29) Some mortal knows some philosopher

(30) At least one man loves at most one woman

(31) At most one woman is loved by at least one man

(32) Most men love Mary

(33) Most men love Agnes

(34) Some men love Mary and Agnes

(35) Some men love Mary or Agnes

(36) John knows and respects all philosophers

(37) John knows or respects all philosophers

Under their respective dominant normal readings, (32) and (33) together entail (34), and (36 entails (37).

Consider now also

(38) At least one man loves Mary or at most two men love Mary

(39) At least one man loves Mary and at most two men love Mary

(40) It is false that at least one man loves Mary

(41) It is false that at least one man loves Mary or it is false that at most two men love Mary

(42) It is false that:it is false that at least one man loves Mary or it is false that at most two men love Mary

(43) Most men love Mary and most men love Agnes

(44) It is false that most men love Mary and most men love Agnes

(45) Most men love Mary or most men love Agnes.

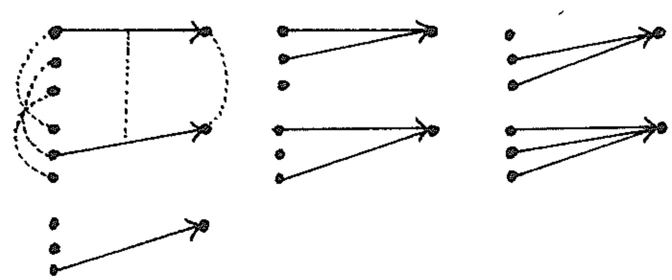
Now, under the dominant normal readings of (13)-(45), we have that: (13) and (14) together entail (15); (16) entails (15); (17) entails (16); and (18) entails (17); (13) and (19) together entail (20); (20) entails (21); (22) entails (14); (23) and (24) together entail (25); (23), (24), and (26) together entail (27); (25) and (28) together entail (29); (31) entails (30) but (30) does not entail (31); (39) entails (1), (4), and (38); (38) and (40) together entail (4); (40) entails (41); (1) entails (38); (39) and (42) inter-entail each other; (43) entails each of (32), (33), (34), and (45).

We will shortly exhibit some of the inferencing machinery that yields various of the above entailments. But first we examine the event diagrams that represent various of the sentences (1)-(43). In the following examples, for the sake of simplicity of illustration, we adopt an index functin that yields fairly small point banks:

The sentence

(1) At least one man loves Mary

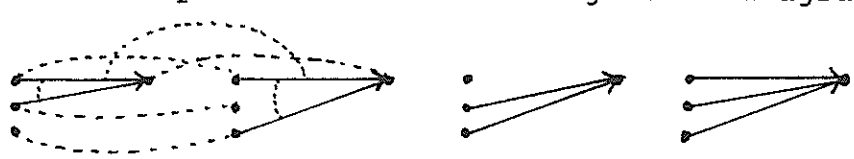
under its dominant normal reading and under an index function that assigns 3 to the noun phrase "at least one man" and assigns 1 to the noun phrase "Mary", corresponds to the following event diagram:



The sentence

(2) At least two men love Mary

under its dominant normal reading and under the above index function corresponds to the following event diagram:



(3) At most one man loves Mary,

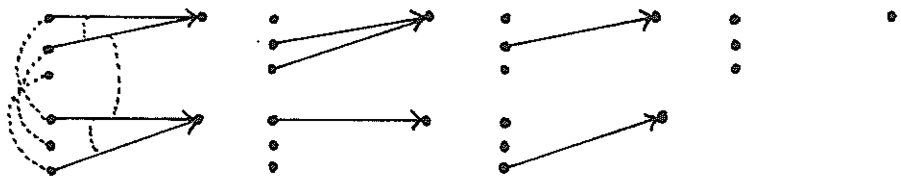
under its dominant normal reading,

is represented by the following event diagram:

(4) At most two men love Mary,

under its dominant normal reading,

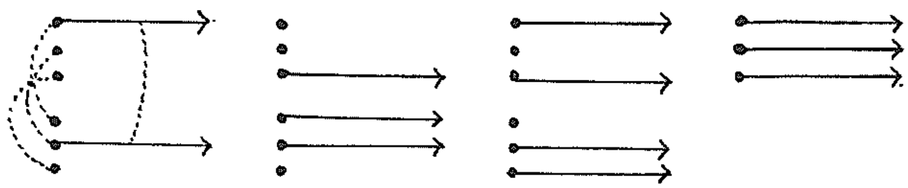
is represented by the following event diagram:



(5) At least one man loves,

under its dominant normal reading,

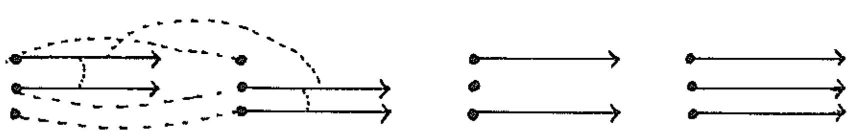
is represented by the following event diagram:



(6) At least two men love,

under its dominant normal reading,

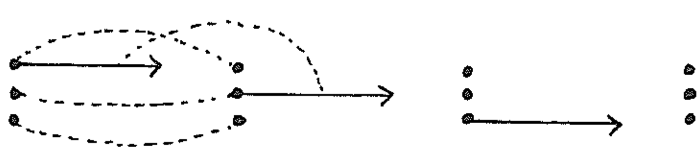
is represented by the following event diagram:



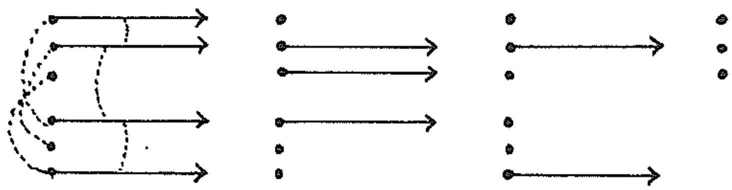
(7) At most one man loves,

under its dominant normal reading,

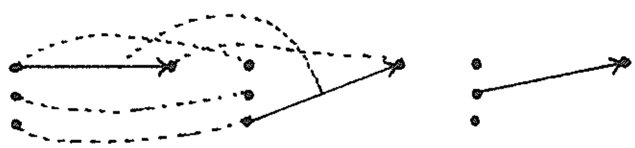
is represented by the event diagram:



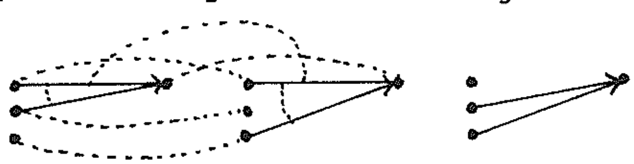
(8) At most two men love,
under its dominant normal reading,
is represented by the event diagram:



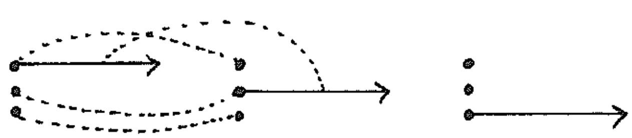
(9) Exactly one man loves Mary,
under its dominant normal reading,
is represented by the event diagram:



(10) Exactly two men love Mary,
under its dominant normal reading,
is represented by the event diagram:



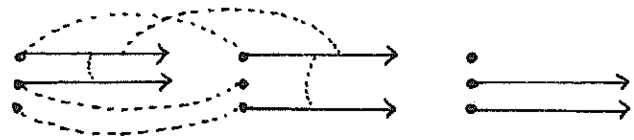
(11) Exactly one man loves,
under its dominant normal reading,
is represented by the event diagram



(12) Exactly two men love,

under its dominant normal reading,

is represented by the event diagram



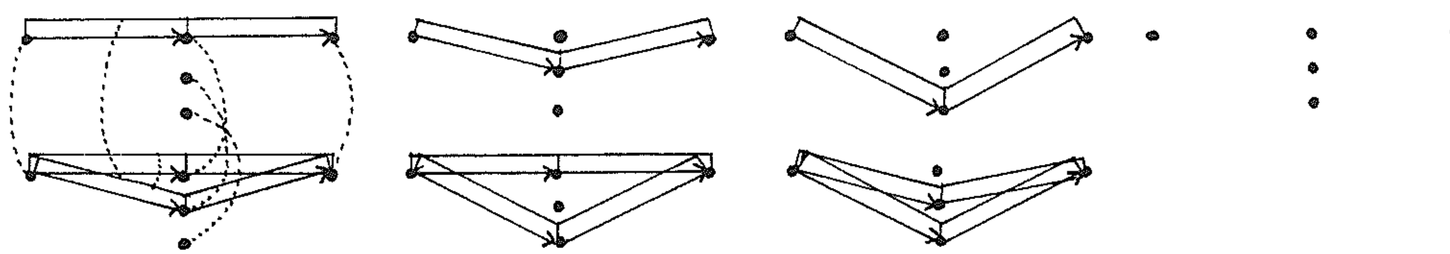
(13) Mary is a waitress

under its dominant normal reading,

is represented by the event diagram

(14) John gave at most two books to Mary

under its dominant normal reading, namely that expressed also by "John gave each of at most two books to Mary", is represented by the event diagram



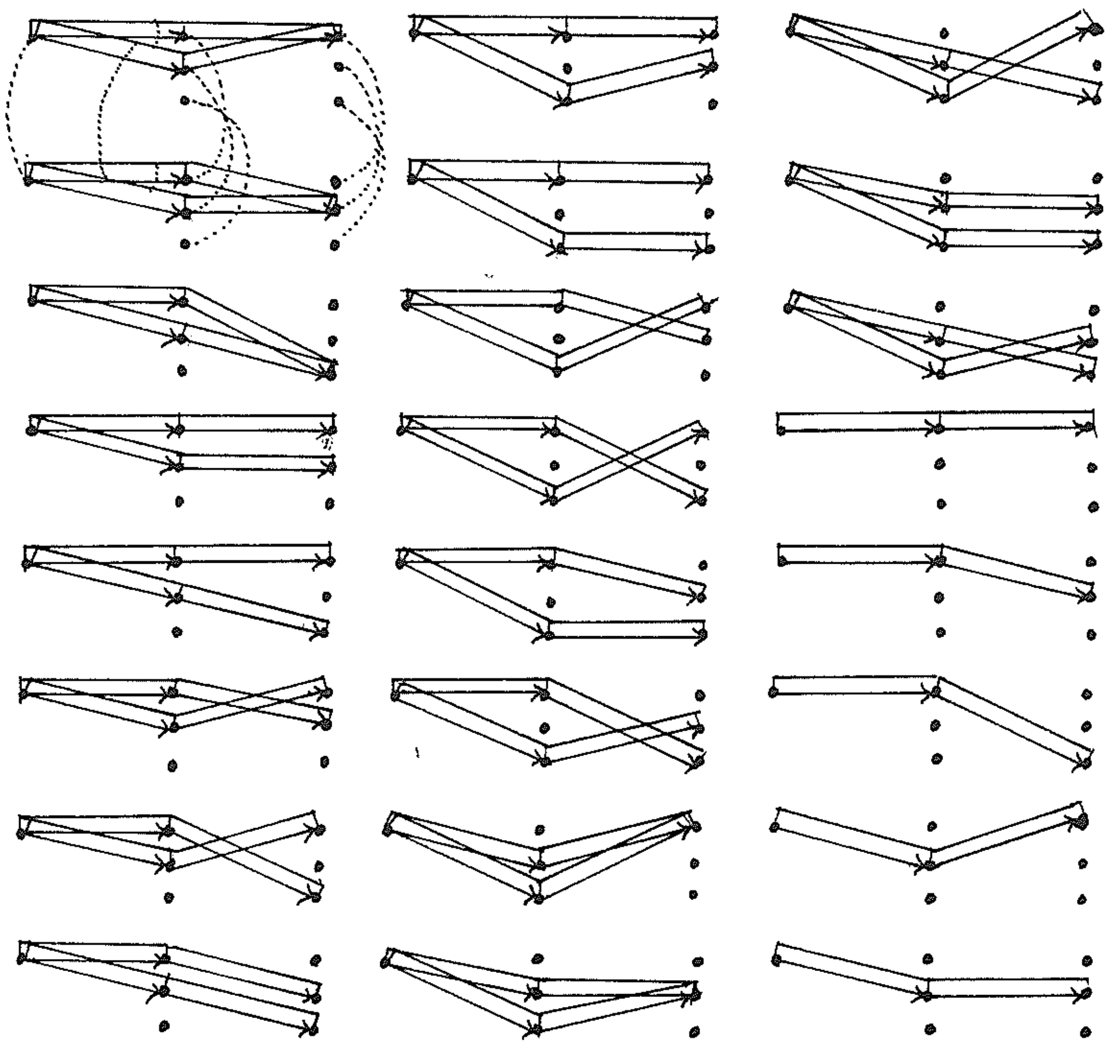
(15) John gave at most two books to a waitress

under one of its normal readings,[39.2] namely that also expressed

Note 39.2 There are other normal readings possible, such as those expressed by:

15(a) John gave a total of at most two books (collectively) to waitresses (but could have given books to non-waitresses as well)
15(b) There is some waitress to whom John gave at most two books

by: John gave each of a total of at most two books and each of those were given by John to the same waitress, is represented by the event diagram:



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(but other waitresses possibly to whom he gave more than two books)
15(c) John gave a total of at most two books (to anyone or anything) and those were given to one particular waitress
15(d) John gave a total of at most two books (to anyone or anything) and those were given (collectively) to waitresses

We note that (15) would have a different event diagram under each of these readings.

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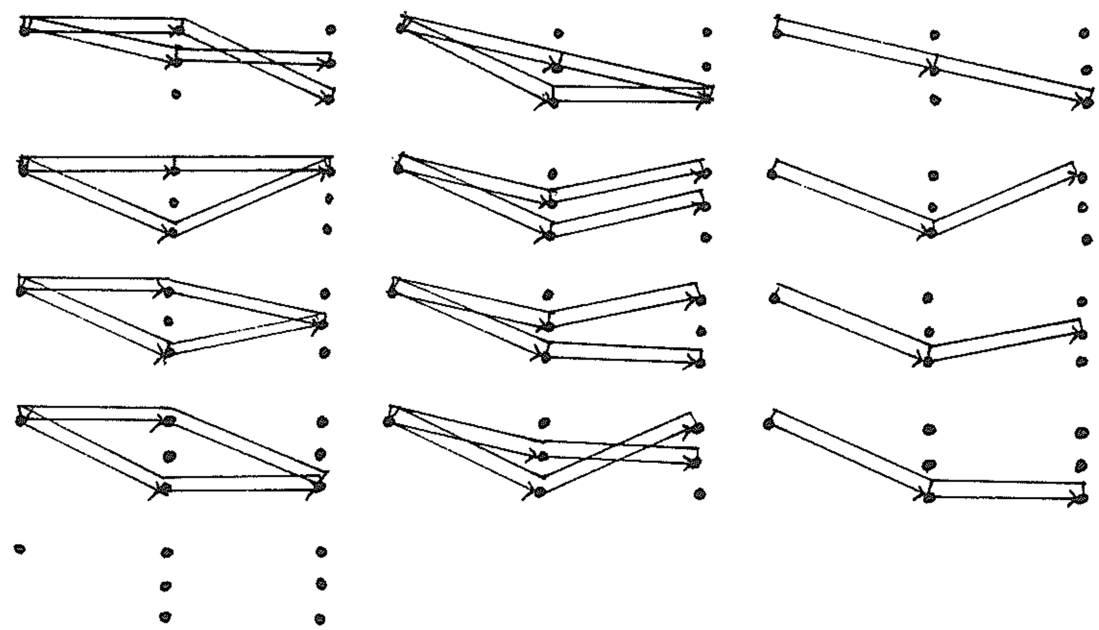
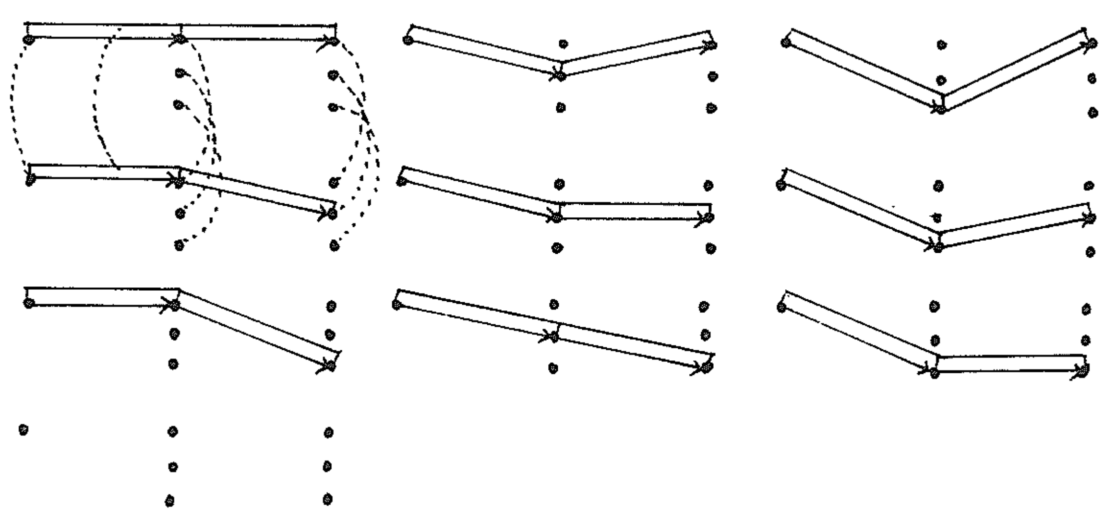


(16) John gave at most one book to a waitress

under one of its normal readings,[39.3] namely that also expressed by: John gave a total of at most one book and that was given by John to exactly one waitress, is represented by the event diagram:



Note 39.3. As with (15) there are other possible normal readings of (16) as noted in Note 39.2.

(17) John gave at most one book

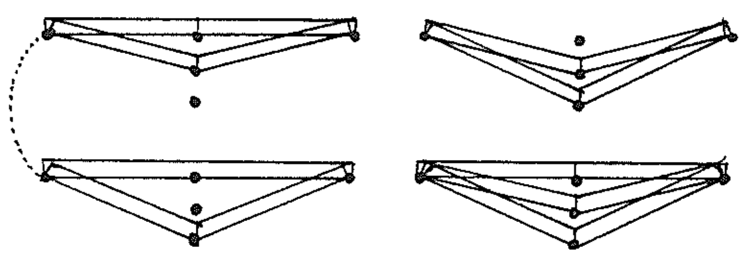
under its dominant normal reading is represented by the event diagram:

(18) John gave no books

under its dominant normal reading is represented by the event diagram:

(19) John gave at least two books to Mary

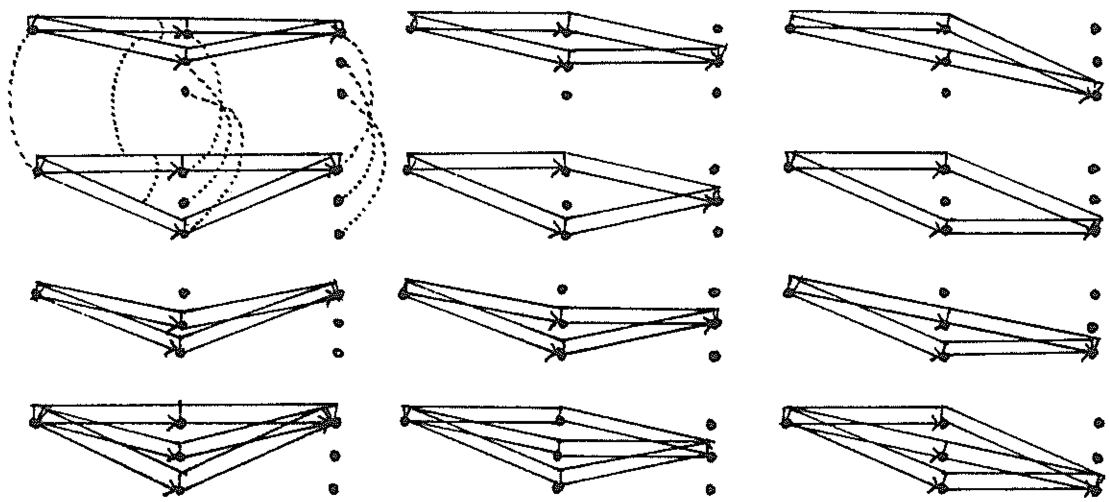
under its dominant normal reading is represented by the event diagram:



(20) John gave at least two books to a waitress

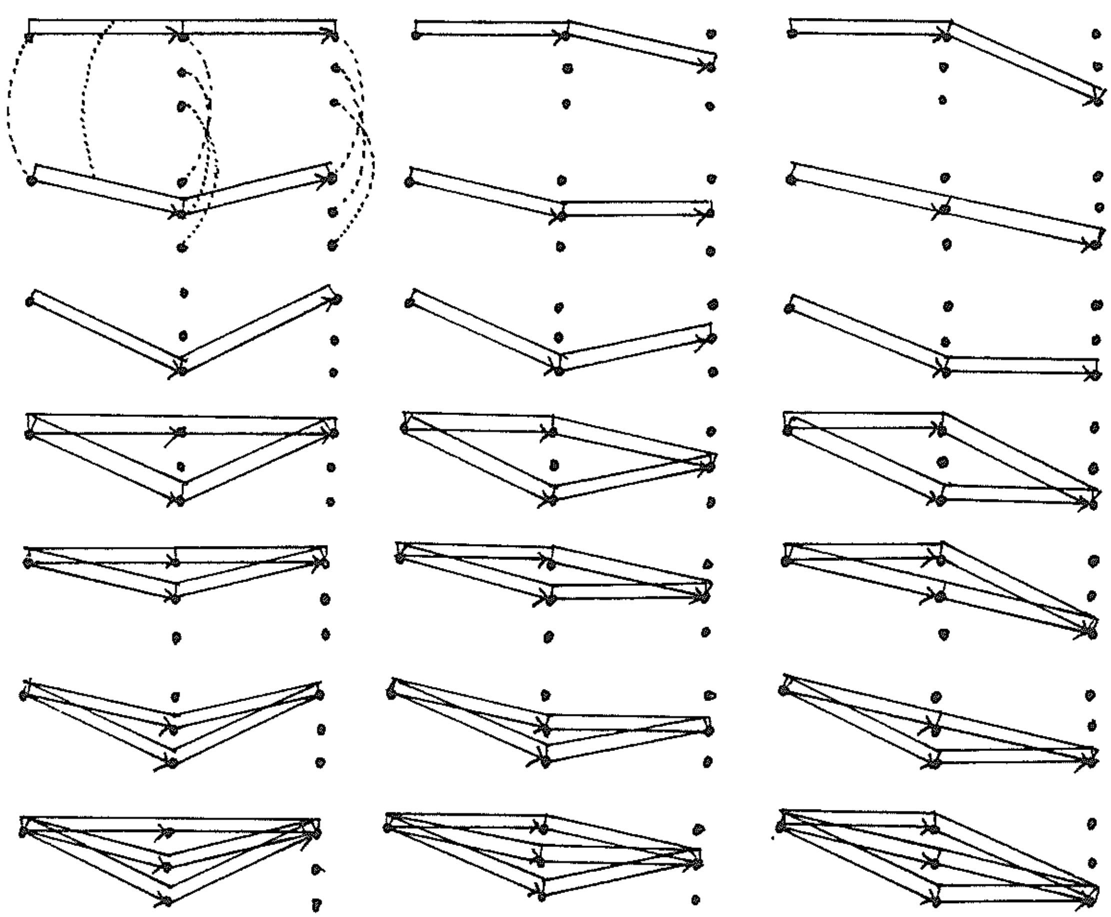
under one of its normal readings, namely that also expressed by:

John gave each of at least two books to the same waitress and did not give any books to any other waitress, is represented by the event diagram:



(21) John gave at least one book to a waitress

under one of its normal readings, namely that also expressed by: John gave each of at least one book to the same waitress, and did not give any books to any other waitress, is represented by the event diagram:

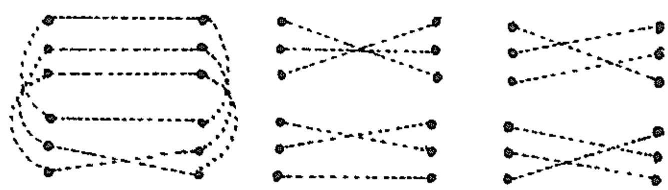


(22) John gave no books to Mary

under its dominant normal reading is represented by the event diagram:

(23) All men are mortal

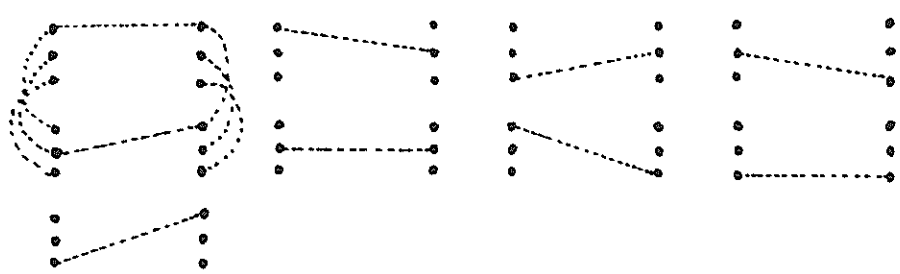
under its dominant normal reading is represented by the event diagram:



We contrast this with

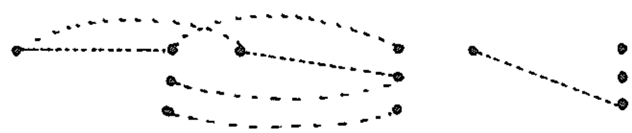
(23a) A man is mortal

under its dominant normal reading is represented by the event diagram:



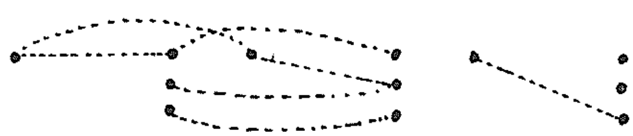
(24) Socrates is a man

under its dominant normal reading is represented by the event diagram:



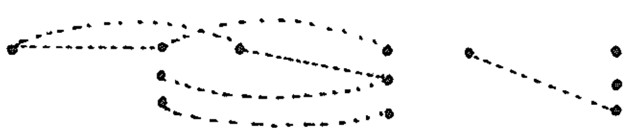
(25) Socrates is mortal

under its dominant normal reading is represented by the event diagram:



(26) Socrates is a philosopher[39.4]

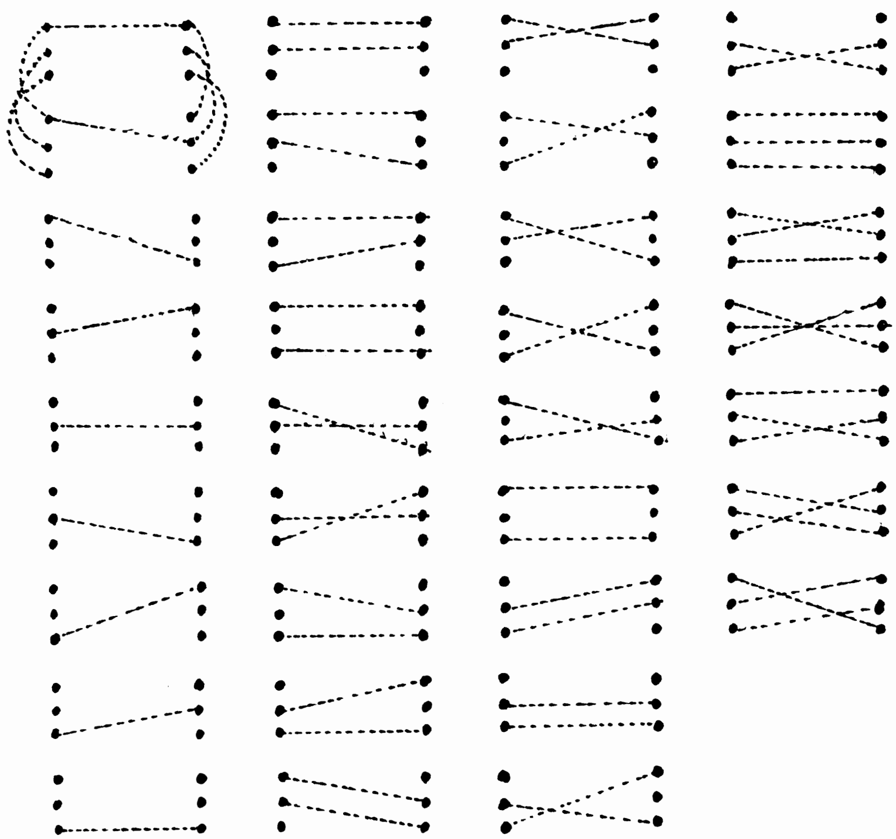
under its dominant normal reading is represented by the event diagram:



Note 39.4. The EDs of (24),(25),(26) are the same when isolated from the other EDs in an RBSN. When they are so linked to other EDs, the difference between (24),(25),(26) is constituted by the fact that the dotted line linkages connecting points representing men to the points of other EDs is different from the dotted line linkages connecting points representing philosophers and mortals to the points of those other EDs.

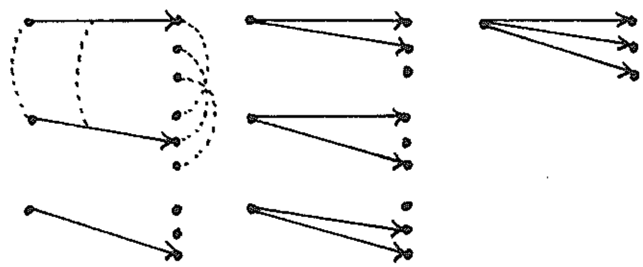
(27) A philosopher is mortal

under its dominant normal reading is represented by the event diagram:



(28) Socrates knows a philosopher

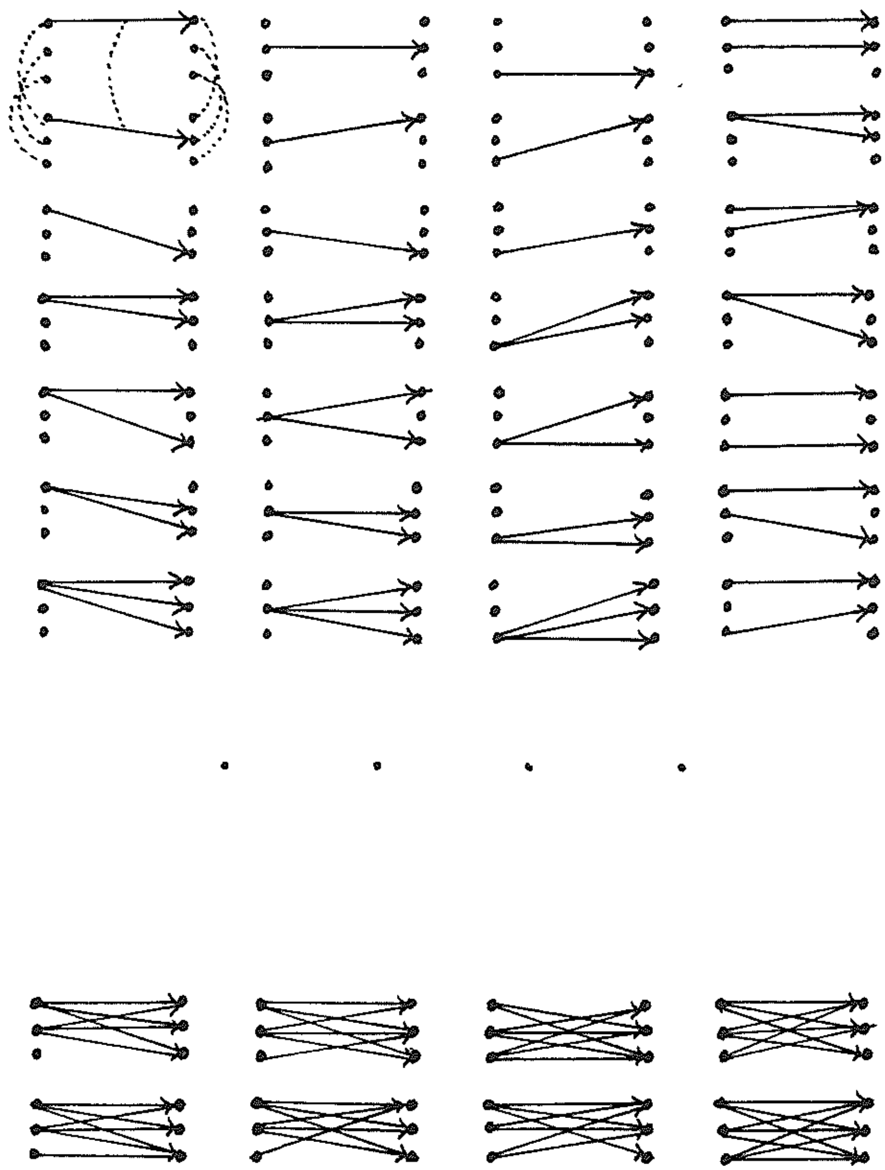
under its dominant normal reading is represented by the event diagram



(29) Some mortal knows some philosopher

under its dominant normal reading

is represented by the event diagram:

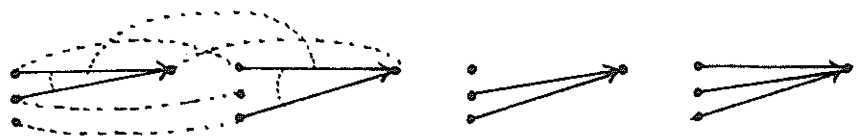


39.1

Note 39.1 There are 2^{3x3} = 512 EPDs in this event diagram.

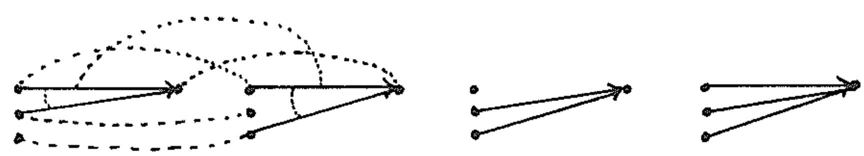
(30) Most men love Mary,

under its dominant normal reading is represented by the event diagram



(31) Most men love Agnes,

under its dominant normal reading is represented by the event diagram



39.2

Note 39.2 For completeness we indicate EDs for both (30) and (31) although, as is clear by inspection, the ED representing (31) is the same as that representing (30). However, in an RBSN containing both, that the points representing Mary and Agnes are not necessarily regarded as representing the same element, is indicated by the absence of a dotted line between those points. See page .

Extended Event Diagrams

The event diagrams we have thus far illustrated are called basic insofar as the only logical expressions whose structure is represented by them are the copula and determiners[40] (signalled by the word-strings "at least one", "at least two", "at most one", "at most two", "exactly one", "exactly two", "a", "John", "Mary", "Agnes"). There are also further types of logical expressions whose representation within RBSNs requires that we extend the diagrams in various ways. We will discuss the treatment of various such further logical expressions, namely those that would occur in readings of word-strings involving conjoined and disjoined noun phrases (e.g., "Mary and Agnes", "Mary or Agnes"), conjoined and disjoined verb phrases, (e.g., "knows and respects", "knows or respects"), and sentential connectives (e.g., "John loves Mary and John knows Agnes", "John loves Mary or John knows Agnes", "If John loves Mary, then John knows Agnes", "John does not love Mary", "John loves Mary if and only if John knows Agnes"). There are also further sorts of

Note 40. As treated in ATR, all logical expressions are expressions of an underlying representation language in which syntactic representations of natural language word-strings are formulated; a natural language word-string is regarded as a "siqnal" which, in conjunction with the context-of-utterance in which the word string is produced, cues an intended or "normal" reading of that word-string relative to that context-of-utterance, of which the syntactic representation of that word-string is a part. In some cases, like "at least one", the signal is explicit; in others, like "John", the signal is implicit - that is, the syntactic representation of "John" contains an "individuator morpheme"--which has no explicit marking in the word "John", and which is interpreted in the semantic theory as a function that maps sets into individual elements, so that the denotation of "John" turns out to be represented by a single point in EDs.

logical expressions that we do not treat here but which can readily be handled by suitable extensions of our diagrams. These would include readings of word-strings involving comparative constructions (e.g., "John is taller than Mary"), temporal constructions (e.g. "John always hits Henry", "John used to hit Henry", "John sometimes hits Henry", "Whenever John hits Henry, Henry cries"), modal constructions (e.g., "It is possible that John hit Henry", "John's hitting Henry necessitated Henry's hitting John."[41]

Note 41. The general theory developed in ATR provides suitable denotations for such expressions in the form of readings for them that are normal with respect to usual context of-utterance. Techniques for representing them within RBSNs have been worked out to a degree sufficient to support the network specification of entailment relations among sentences incorporating such expressions.

A great variety of further grammatical constructions, beyond those illustrated here, can also be represented within RBSNs. These include, besides the temporal and modal adverbials indicated here, also further modifier constructions of diverse sorts, adjectival, adverbial, phrasal, clausal, extensional, and intensional. We remark in this footnote on one type of modifier construction, referred to in ATR as the differentiated relative, which is distinguished from the ordinary (restrictive and non-restrictive) relative in the sense that it has different entailment properties. The differentiated relative is (typically but not exclusively) signalled in English by the preposition "of", as occurs in phrases like "the vase of flowers", "the mother of Henry's friend", "the square of 3", "the sum of 3 and 5", and so on.

There are two interesting consequences of the fact that differentiated relative constructions can be represented within RBSNs:

(i) Certain types of English sentences such as "Some friend of every man and some enemy of every woman know each other" cannot be properly formalized within 1st order logic unless one extends that logic to allow for non-linearly ordered (i.e., "branching") quantifiers, or, alternatively, one formalizes them within second order logic. This was first noted by Henkin and subsequently analyzed by Hintikka, who regarded such sentences as arguing for the richness of natural language constructions. The point being made here is that such constructions, hence sentences

Representing Conjoined and Disjoined Noun Phrases

As a simple example of one of these further logical expressions, we consider "and" as it enters into the construction of the conjoined verb phrases "Mary and Agnes" in the sentence

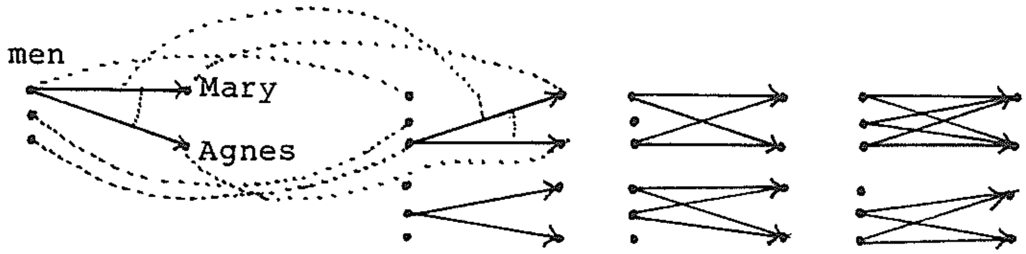
(34) Some men love Mary and Agnes

which, under its dominant reading, is represented by the event diagram:[42]

like the preceding example, are very easily representable within RBSNs as described in this paper and suggest that such RBSNs have considerable "expressive power".

(ii) The differentiated relative can be used to represent mathematical functions within RBSNs (e.g., "the sum of 2 and 3") thereby providing graphic models of diverse axiomatically given mathematical theories within a uniform graphic vocabulary (i.e., a vocabulary whose basic elements are points, arrow traces, and dotted lines, as described in the body of this paper). These graphic models can be regarded as "most general" or "most neutral" models of those mathematical theories insofar as their graphic structures reflect only the syntactic structure of the axioms defining those theories, imposing no further features onto those models than is directly warranted by that syntactic structure.

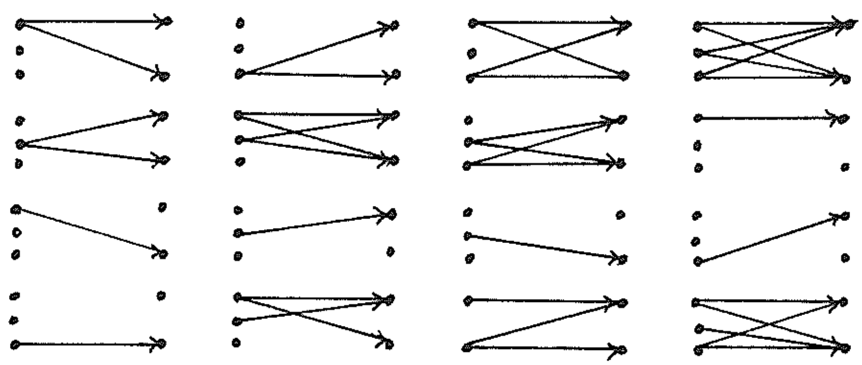
Note 42. Even though the points representing the elements Mary and Agnes in the above event diagram are indistinguishable as points, they are distinguished within an RBSN containing the above ED by virtue of the fact the configuration of dotted lines connecting other points in other EDs representing other sentences to the point representing Mary is in general different from the configuration connecting points in other EDs to the point representing Agnes. See note 39.4.



As another simple example of one of these further logical expressions, we consider "or" as it enters into the construction of disjoined verb phrases "Mary or Agnes" in the sentence

(35) Some men love Mary or Agnes

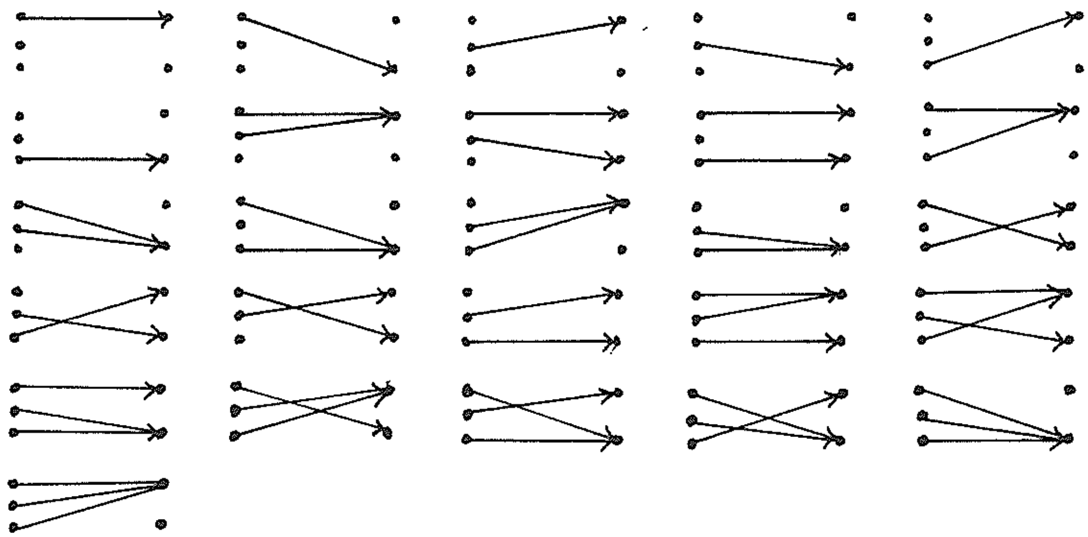
Under that one of its normal readings in which "or" in (35) is understood in its *inclusive* sense, (35) is represented by the event diagram:



. . . . . 42.1

Note 42.1 We use dots ... to indicate that we have exhibited only part of the ED in question.

Under that one of its normal readings in which "or" in (35) is understood in its exclusive sense, (35) is represented by the event diagram:



2.7 Complex EDs and the Extended Entailment Form

The EPDs and arrow paths we have thus far described are simple (in a sense to be defined more precisely below) and are distinguished here from complex arrow paths and complex EPDs, to be introduced in this section. EDs whose constituents are simple EPDs and which are of the type thus far described in this paper, are called simple EDs, and are distinguished here from complex EDs also to be introduced in this section. Complex EDs are required to represent various further natural language constructions beyond those we have thus far considered. Two such constructions requiring complex EDs for their representation are verb phrase compounds using and and or, and sentential combinations of sentences using the binary sentential connectives and, or, if, then, and if and only if.

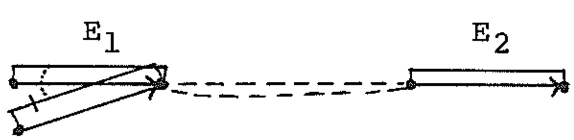
Given m EPDs $E_1,\ldots,E_m$, a complex EPD with constituents $E_1,\ldots,E_m$ is an array of all possible weakly braced arrow paths, that can be formed by imposing a weak brace onto the arrow paths of $E_1,\ldots,E_m$ in such a way that, for each $1 \leq i \leq m$, the i^{th} major constituent of p is an arrow path of E_i.

For example, given the EPDs E_1, E_2:

(a)

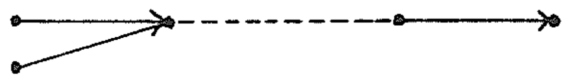
then the following is a complex EPD with constituents E_1, E_2:

(b)



We note that, in this example, E_1 and E_2 can be linked by dotted lines in a variety of ways, which remain intact when the dashed lines of the weak brace are imposed onto them.

However, for visual clarity, in illustrating complex EPDs and complex EDs for the arrow traces that occur in our examples below, we diagram (b) in the examples below more compactly as:

(c) 

Thus, generally, all complex EPDs whose constituents occur within a given RBSN are formed by imposing dashed lines joining those constituent EPDs in the manner of (b).

Two complex EPDs A, B with constituents $A_1,\ldots,A_m$, and $B_1,\ldots,B_n$ respectively, are <u>similar</u> if and only if m = n and, for each $1 \leq i \leq m$, A_i is similar to B_i.

Given m EDs $E_1,\ldots,E_m$, a <u>complex ED with constituents $E_1,\ldots,E_m$</u> is an array of all possible complex EPDs with constituents $E_1',\ldots,E_m'$ which themselves are constituents of $E_1,\ldots,E_m$ respectively.

If the m EDs occur within a given RBSN, that complex ED with those EDs as constituents already exists within that RBSN. However, for display purposes we exhibit complex EDs in standard form.

<u>Representing Compound Relations (Verb Phrases) Within RBSNs</u>

One use of complex EDs is in representing compound verb phrases such as those occurring in:

John knows and respects all philosophers

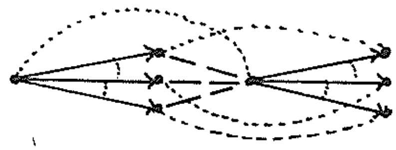
John knows or respects all philosophers (in the inclusive sense of "or", and in the exclusive sense of "or")

John knows but does not respect all philosophers.

Using these diagramming conventions, we can represent the denotation of the conjoined relation of (36), and the denotation of the disjoined relation of (37) as follows:

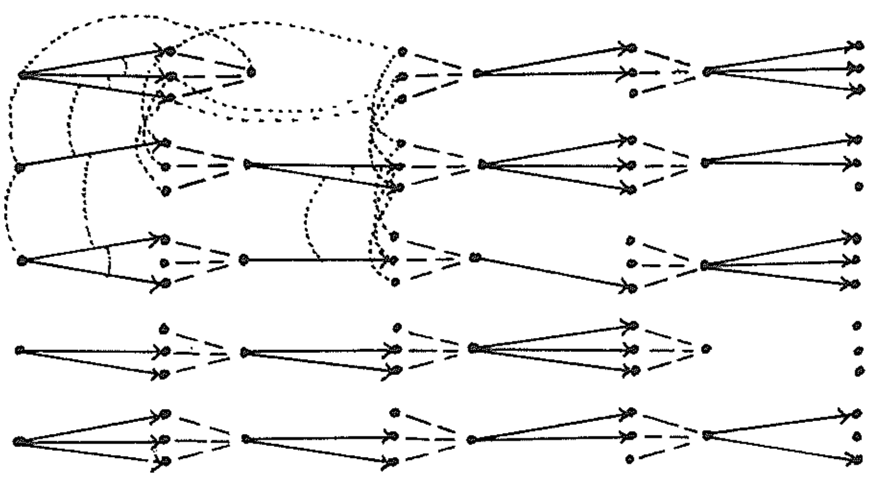
(36) John knows and respects all philosophers,

under its dominant normal reading, is represented by the event diagram:



(37) John knows or respects all philosophers,

under its dominant normal reading (relative to which "or" has the inclusive sense) is represented by the event diagram:



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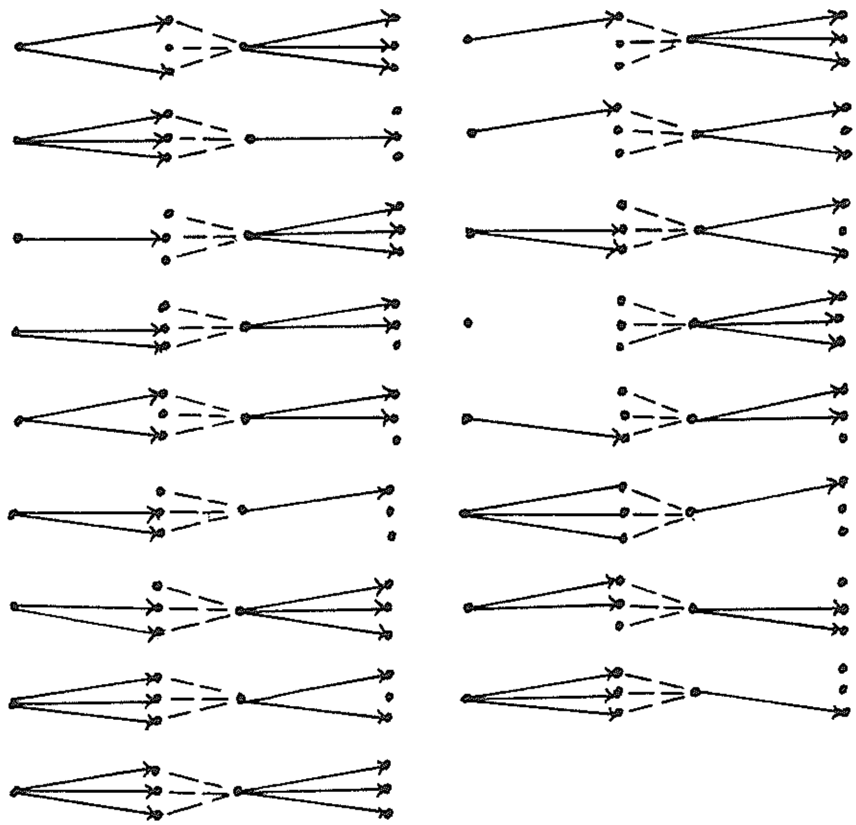
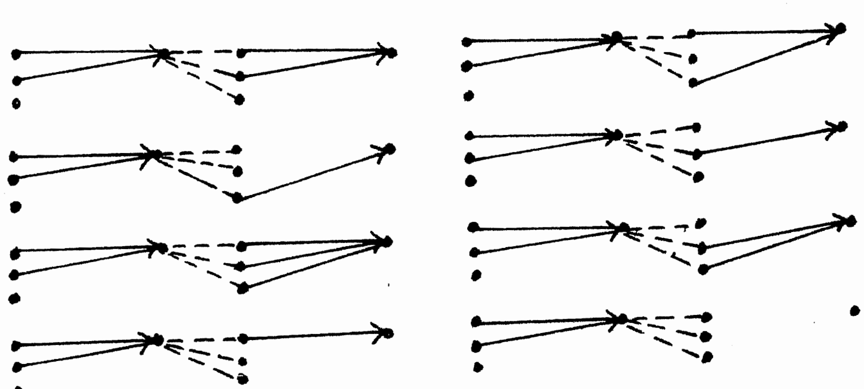


Sentential Connectives

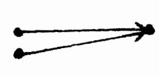
In this section we discuss the diagrammatic representations of the logical relations corresponding, respectively, to the sentential connectives of inclusive and exclusive disjunction, conjunction, negation, implication, and equivalence. In order to describe these representations we will need to introduce some further notions (a) - (d) below pertaining to diagrams:

(a) Given a complex ED E, we obtain a reduction of E by ~~(i)~~ deleting from E those constituent complex EPDs with constituent EPDs (simple or complex) that are similar ~~but~~ non-identical, (ii) deleting, within any constituent complex EPD E' of E, all of the immediate constituent EPDs of E' (simple or complex) that are identical to an earlier constituent EPD within E', and (iii) deleting from E all repeated constituent EPD; and we obtain a radical reduction of E if, in addition, we delete, within any constituent (simple or complex) EPD E' of E, all of the immediate constituent EPDs E'' of E' (simple or complex), that are such that the result of replacing E'' in E' by any EPD similar to E'' is also in E.

For example, the ED:



has the radical reduction:



(b) Let A, B be EDs. Then $A \mathring{\times} B$ is a reduction[40.2] of a complex ED E such that the immediate constituent EPDs of E are just those complex EPDs whose first constituent is an EPD from A and whose second constituent is an EPD from B.

(c) Let A be an ED. Then A^c is a reduction[40.2] of an ED E such that the immediate constituent EPDs of E are just those EPDs which are similar to some EPD in A and not an immediate constituent of A.

(d) Let A, B be EDs. Then $A \mathring{\cup} B$ is a reduction[40.2] of an ED E such that the immediate constituent EPDs of E are just those EPDs which are either immediate constituent EPDs of A or are immediate constituent EPDs of B.

In the following definitions, we continue to employ the convention of writing "e'" to indicate the syntactic representation of the natural language word-string e which e receives under the dominant reading of e, that is, the most usual reading of e. Also we designate the ED corresponding to e under the reading (e',s) by E[s[e']].

<u>Note 40.2</u>. We could, alternatively, have omitted the phrase "a reduction of" in the definitions of x, and c, hence rendered the resulting EDs constructed by application of x, c, and $\mathring{\cup}$ (which is defined in terms of x and c) to component EDs. This alternative approach, while conceptually simpler, would have given rise to very large EDs containing many redundant arrow paths, and would have obscured the way that the operations x, c, and reflect the sentential operations of conjunction, negation, and disjunction, respectively. In particular, the notion of reduced EDs makes it possible to obtain the De Morgan relationships:

$$A \mathring{\cup} B = (A^c \mathring{\times} B^c)^c$$

$$A \mathring{\times} B = (A^c \mathring{\cup} B^c)^c$$

Let a, b be natural language sentences, let s be a semantic theory, and let (D,f) be an interpretation in s. Then:

(i) $E[s[(a \text{ or } b)']] = E[s(a')] \mathring{\cup} E[s(b')]$

(ii) $E[s[(a \text{ and } b)']] = E[s(a')] \mathring{\times} E[s(b')]$

(iii) $E[s[(\text{not } a)']] = (E[s(a')])^c$

(iv) $E[s[(\text{if } a, \text{ then } b)']] = (E[s(a')])^c \mathring{\cup} E[s(b')]$

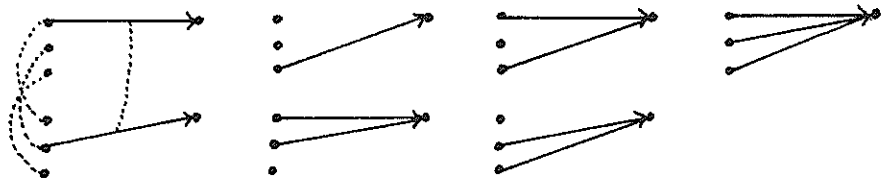
(v) $E[s[(a \text{ if and only if } b)']] = (E[s(a')] \mathring{\cup} E[s(b')]) \mathring{\times} (E[s(b')])^c \mathring{\cup} E[s(a')])$

Let us consider some examples:

Recall (from pages 80-82) the ED (1.1) corresponding to (1):

(1) At least one man loves Mary

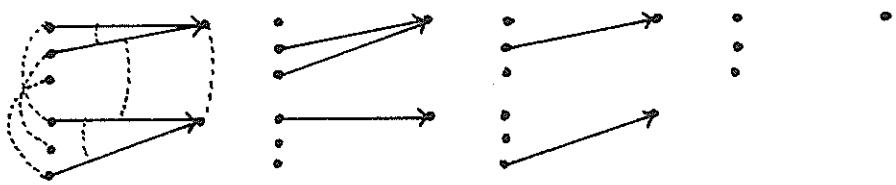
(1.1)



and the ED (4.1) corresponding to (4):

(4) At most two men love Mary

(4.1)

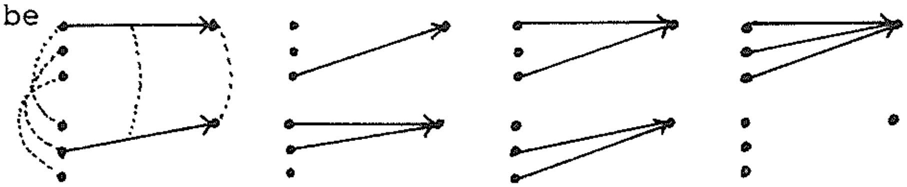


Thus the ED for

(38) At least one man loves Mary or at most two men love Mary

would be

(18.1)

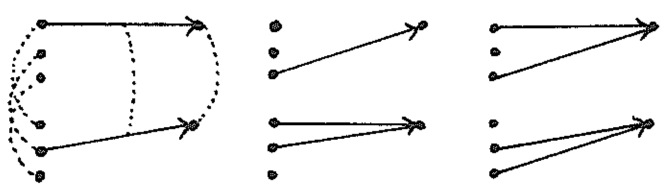


The ED for

(39) At least one man loves Mary and at most two men love Mary,

would then be

(39.1)



The ED for

(40) It is false that at least one man loves Mary

would be

(40.1) 

In the special case of the negations of (1.1), (4.1), and (43.1), below, the ED corresponding to the negation of a sentence, reduces to the set-theoretic complement of the ED corresponding to that sentence.

The ED for

(i) It is false that at most two men love Mary

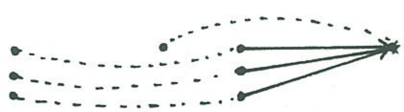
would be

(i.1) 

The ED for

(41) It is false that at least one man loves Mary or it is false that at most two men love Mary

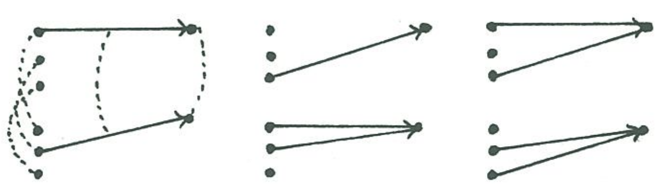
would be

(41.1) 

The ED for

(42) It is false that: it is false that at least one man loves Mary or it is false that at most two men love Mary

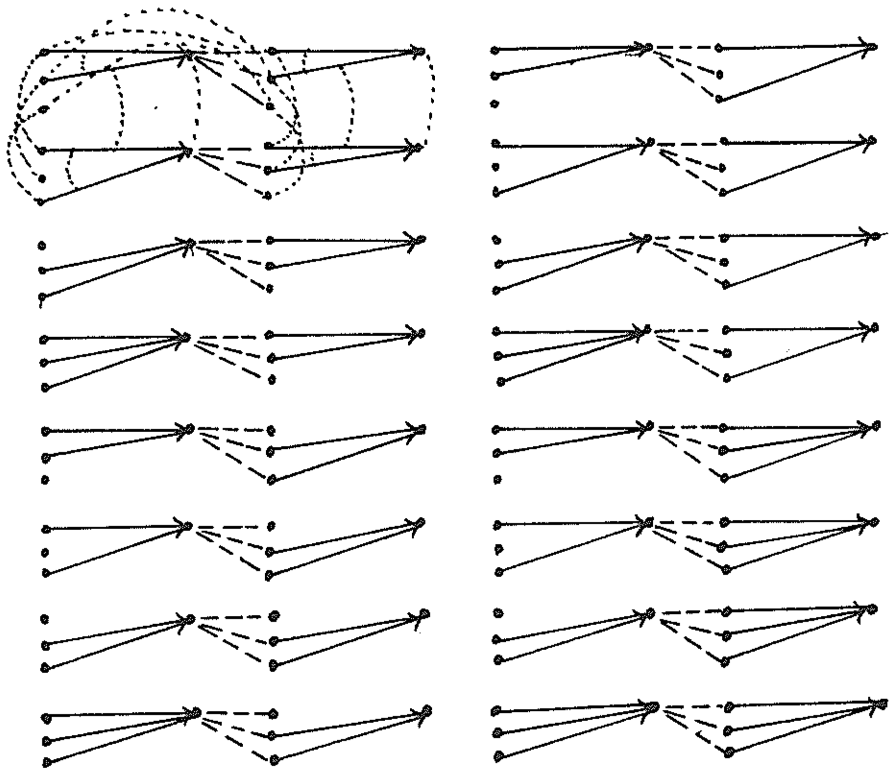
would be

(42.1) 

The ED (43.1) below corresponding to (43) would be:

(43) Most men love Mary and most men love Agnes

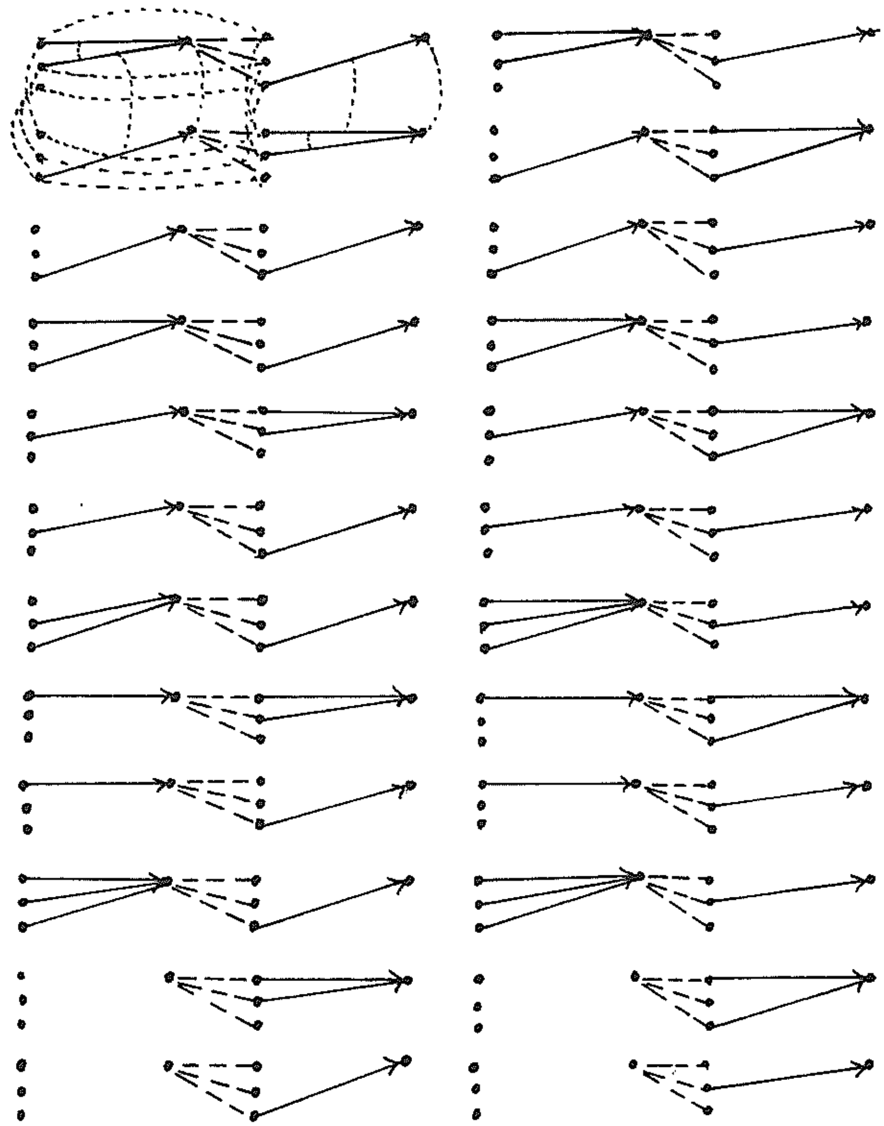
(43.1)



The ED (44.1) (below corresponding to (44) would be:

(44) It is false that most men love Mary and most men love Agnes

(44.1)[43]



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Note 43. We note that the number of possible EPDs with point banks of 3 and 1 element respectively is $2^3 = 8$, and that the number of possible pairs of such EPDs is $8 \times 8 = 64$, and that this is the total number of EPDs of (43.1) and (44.1), namely 16 and 48 respectively.

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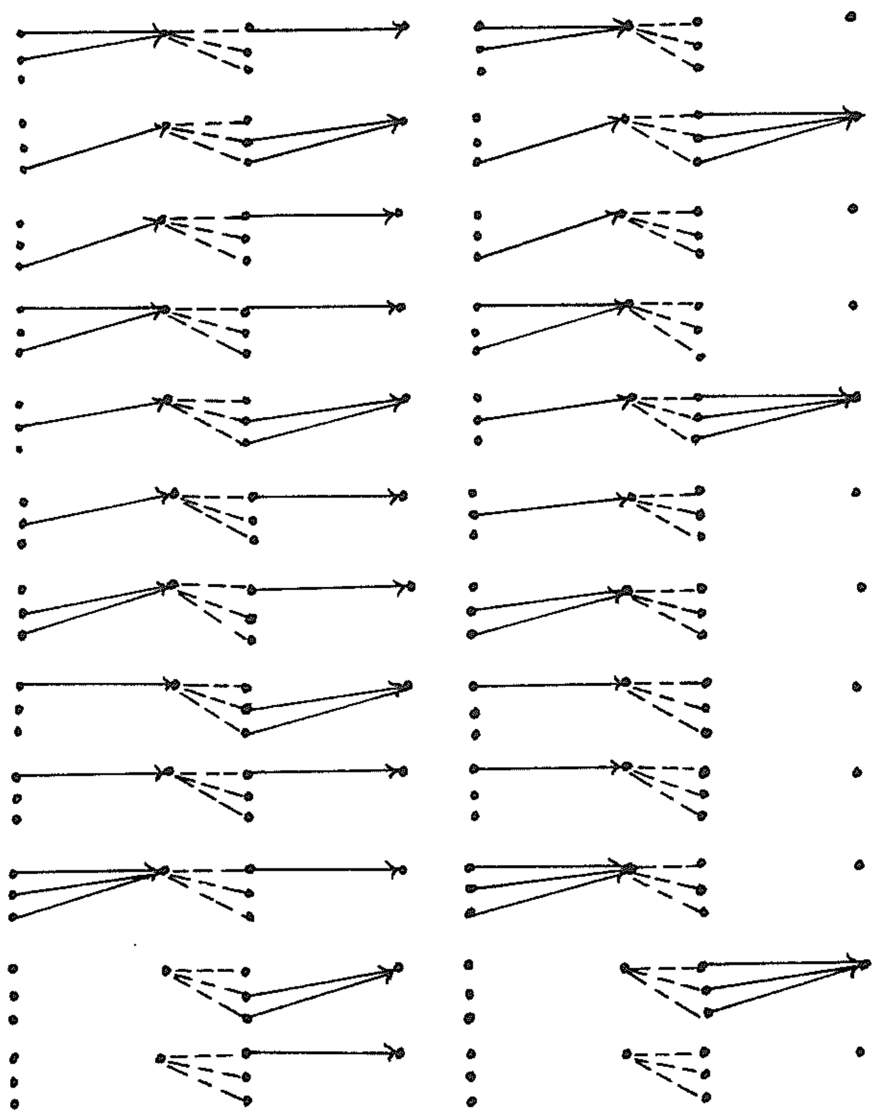
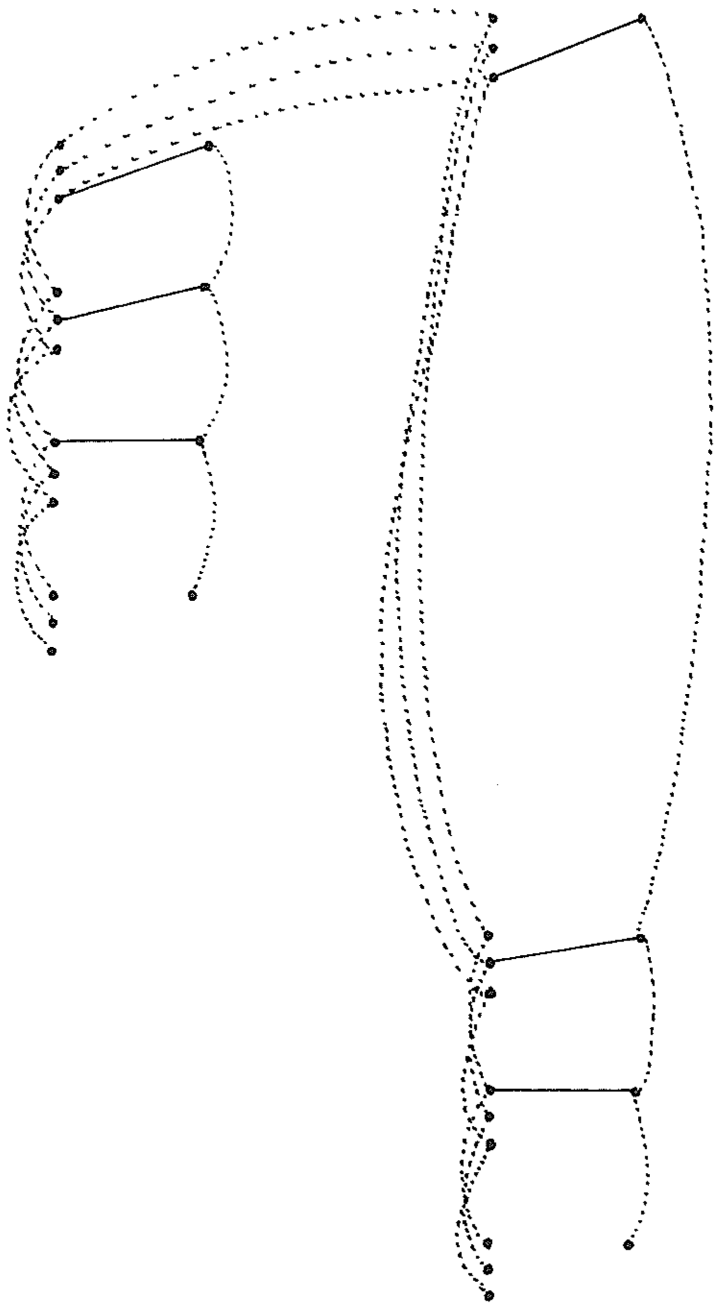


The ED 44.1 is equivalent to the following ED (44.1a) which is a radical reduction of (44.1)

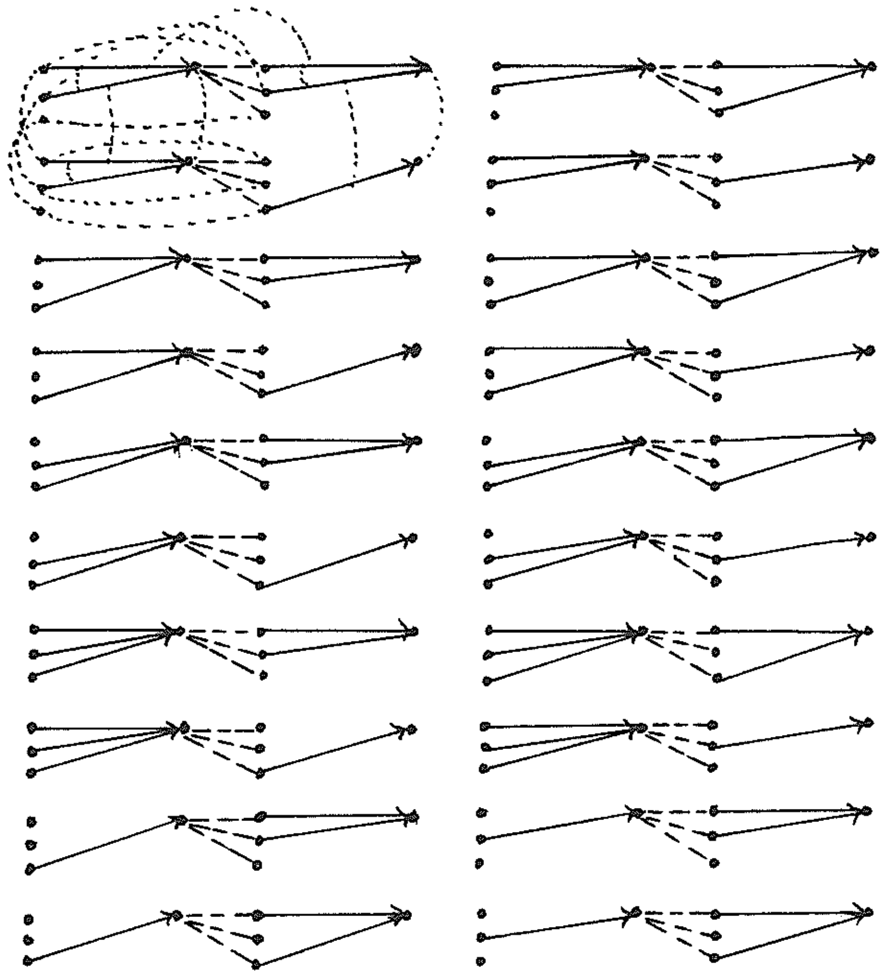
(44.1a)



The ED (45.1) below corresponding to (45) would be:

(45) Most men love Mary or most men love Agnes

(45.1)



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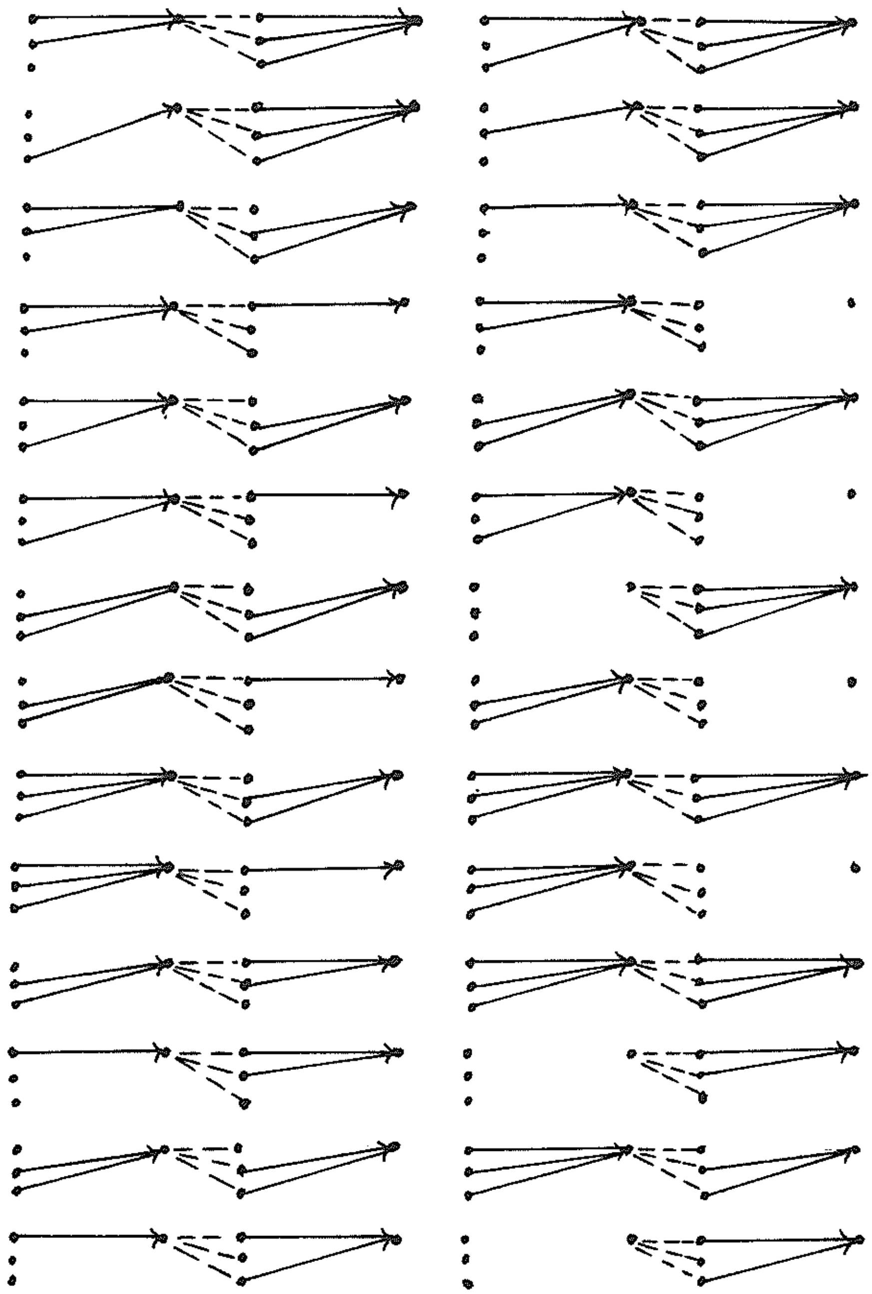
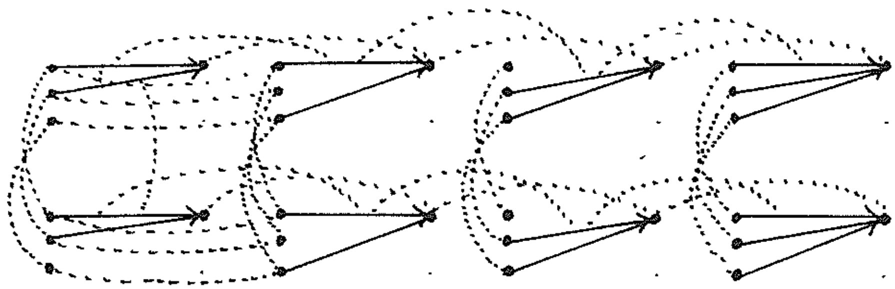


We note that the ED (45.1) is equivalent to the following radical reduction of (45.1):

(45.1a)



because each of the EPDs of the ED of the sentence "Most men love Mary" is paired with each of the EPDs in the closure of the ED of the sentence "Most men love Agnes", and each ofthe EPDs of the ED of the sentence "Most men love Agnes" is paired with each of the EPDs in the closure of the ED of the sentence "Most men love Mary". Since the closures of each of the EDs[43.1] of "Most men love Mary" and of "Most men love Agnes" contains 2^3 = 8 EPDs , the number of EPDs in (45.1) 8x8=64 minus the number of repetitions: 4x4 = 16, which equals 64 - 16 = 48.

Note 43 .1. See examples (30), (31) on page 94 above.

We now state the Extended Entailment Form which allows for the possibility that some or all of the entailing or entailed EDs are complex, and yet covers the case of the Basic Entailment Form, where all EDs are simple, as a special case.

<u>The Extended Entailment Form</u>. Let $ED_1, \ldots, ED_m$, ED_{m+1} be EDs of an RBSN α. Then $ED_1, \ldots, ED_m$ (relative to α) <u>entail</u> ED_{m+1} if and only if, for every consistent choice of immediate constituent (simple or complex) EPDs, $E_1, \ldots, E_m$ one from each of $ED_1, \ldots, ED_m$ respectively, there is a (simple or complex) immediate constituent EPD E_{m+1} of ED_m such that, for every simple constituent EPD E'_{m+1} of E_{m+1}, (if E_{m+1} is simple, then $E'_{m+1} = E_{m+1}$), each of the arrow paths in some reduction set of E'_{m+1} relative to the simple EPDs of ED_{m+1} is a resultant relative to α of some generalized arrow path of the simple EPDs that are constituents of $E_1, \ldots, E_m$.

2.8 Examples of Entailment

In Section 1.4, we discussed a simple special case of entailment involving a single entailing ED which was, furthermore, similar to the entailed ED. The following series of examples illustrates various more complex cases of entailment involving various of the sentences (1) - (45)

Explicit Representation of Resultants

Recall that, from pages 71, 72, if a given arrow path p is an explicit resultant of a given generalized arrow path p^, then p occurs as an actual part of p^. We call this an explicit representation of p as a resultant of p^. For ease in illustration and for perspicuity, it is convenient to represent p implicitly as a resultant of p^ by having the EPD of which p is a part separate from the EPDs in which the generalized arrow path p^ occurs, linking all corresponding arrow traces and points by dotted lines.

In the following diagrams, the sample EDs that are identified as entailed by a given RBSN are implicitly represented. It is clear that the entailed EDs could alternatively have been explicitly represented, that is, they could have been identified in terms of requisite relationships among the arrow paths comprising the EPDs of the interconnected event diagrams comprising the RBSN. Indeed, in most implementations, it would be more economical to allow the entailed EDs to be explicitly represented within an RBSN, for then one could identify just those among all entailments yielded by an RBSN that were of interest in a given application.

The description of an explicit representation of entailed EDs could be readily derived from the preceding description of their implicit representation, though we shall not do so here.

Extension to Three Dimensional RBSNs

For simplicity of description, the present account has been restricted to EPDs, EDs, and the entailment paths among EDs that utilize only the diagrammatic representations of the major[44] thing and relation expressions of sentences rather than including embedded thing and relation expressions as well. For many entailments, such as the ones we have illustrated here, such representation at the "topmost" level of a sentence will suffice. But for other entailments we need to utilize also the diagrammatic representations of more deeply embedded expressions as for example, where a pronominal referent of a major thing-expression was embedded within a further expression, such as within an ordinary or differentiated relative construction which was a subexpression of another major thing-expression.

Thus for general purposes RBSNs need to be extended from the two-dimensional configurations, representing only "topmost" expressions, as described and illustrated above, wherein interconnected EDs can be visualized as being on a plane, to a three dimensional[46] configuration, where the third dimension imparts

Note 44. See Appendix for an elaboration of the underlying grammar.
Note 46. Representation in three dimensions is intended here only as a conceptual aid to visualization: the indicated extensions can of course also be represented within two dimensions.

"depth," that is, diagrammatically represents all of the subexpressions of the sentences whose representations occupy the topmost plane and interconnects the representation of those subexpressions by referential links to each other and to the EDs in the topmost plane. The development of such three-dimensional RBSNs is being currently pursued and the results will be included in an updating of the present paper.

An RBSN can be physically represented in circuitry in a manner where all arrow traces and dotted lines shown in an RBSN are current-carrying physical components, and points in an RBSN are points from which and to which current is carried.

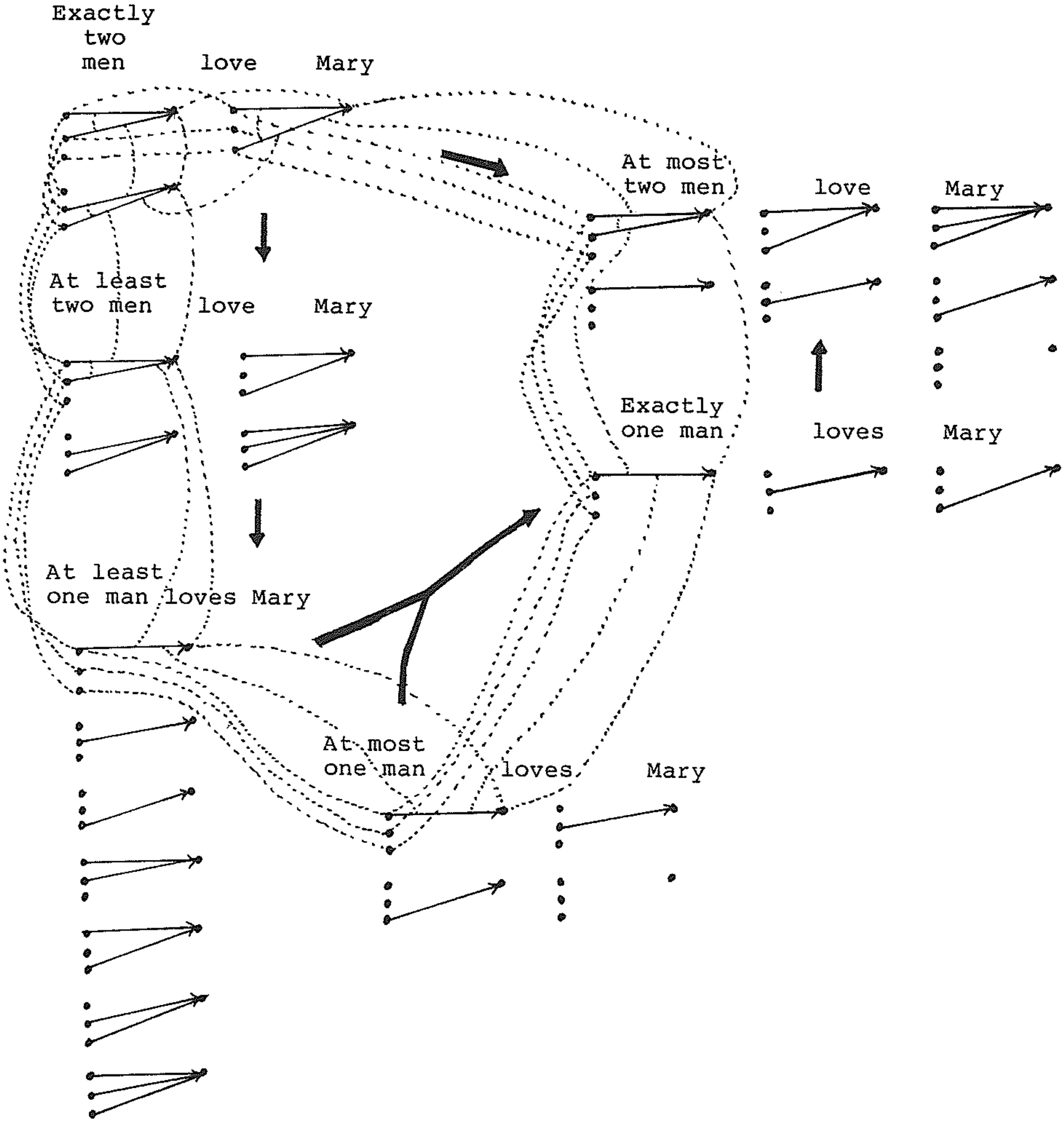
A three-dimensional RBSN, when physically represented in circuitry, could be regarded as a reasonable physical model of deductive functions of the brain that accounts for deductive functions as they apply to disambiguated (i.e. syntactic representations of) natural language word-strings. The model would represent information corresponding to deductive relationsips among linguistic units at the phrase, sentence, and discourse levels as configurations of electrical currents along arrow paths and dotted lines. The model "grows" as new information is entered, and automatically forms referentiality-indicating electrical analogues of the dotted lines used in our diagrams. In the explicit representation of entailments, it "searches" for pathways to determine the entailment patterns that hold among its sentences; in the implicit representation of entailments, it "seeks" to determine whether the conditions that would give rise to such suitable pathways relative to given entailments of interest already exist within the model.

In the following illustrations of entailment, we employ a solid arrow $\Longrightarrow$ to indicate the direction of entailment.

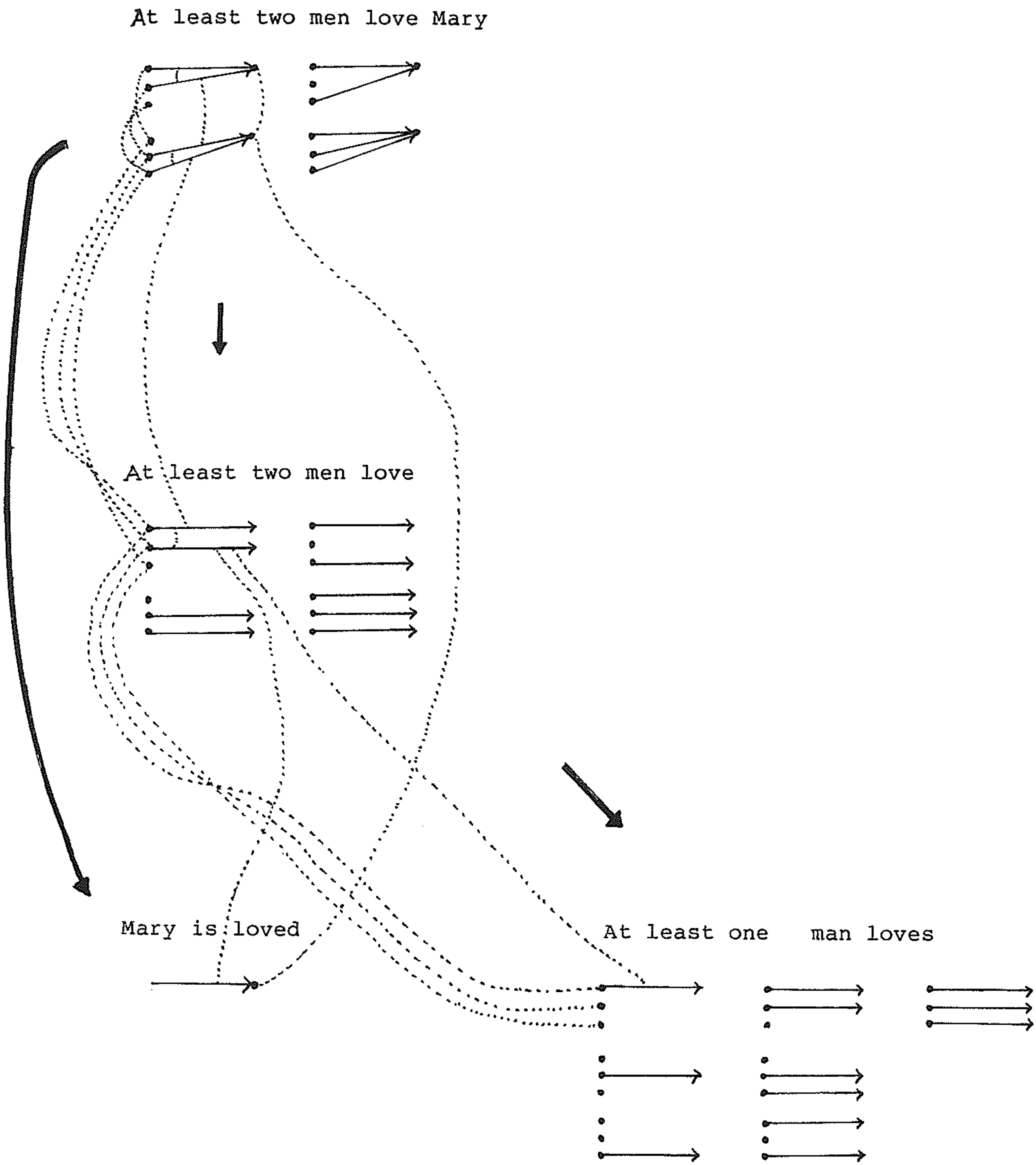
It can be easily verified that the definition of entailment given earlier means that for any given choice of EPDs $E_1,\ldots,E_m$ chosen respectively from entailing EDs $ED_1,\ldots ED_m$ there is a choice E_{m+1} of an EPD in the entailed ED ED_{m+1} and a reduction set E'_{m+1} of ED_{m+1} such that every arrow path p of E'_{m+1} is a resultant of some generalized arrow path $p^\wedge$ of E'_{m+1}.

For simplicity we include only such diagrammatic components in the following examples of entailment as are required to illustrate the given case of entailment. In particular, we continue to employ the simplifications already employed in the display of event diagrams, namely, we usually omit barred arrow paths and dot paths; we include only sufficiently many dotted lines joining points and arrow traces as are required to indicate their general configuration within and among event diagrams; we display only sufficiently many event particular diagrams within a given event diagram as are required to indicate their general character within that event diagram. In addition, regarding entailment, we indicate only one typical choice of EPDs from the entailing EDs, and only a few of the dotted lines joining them with an EPD that they determine from the entailed ED. Moreover, we include only those (if any) explicit resultants of given arrow paths that would actually be required in a given entailment. (Recall that all explicit resultants of arrow paths within a given RBSN would also occur within that RBSN.)

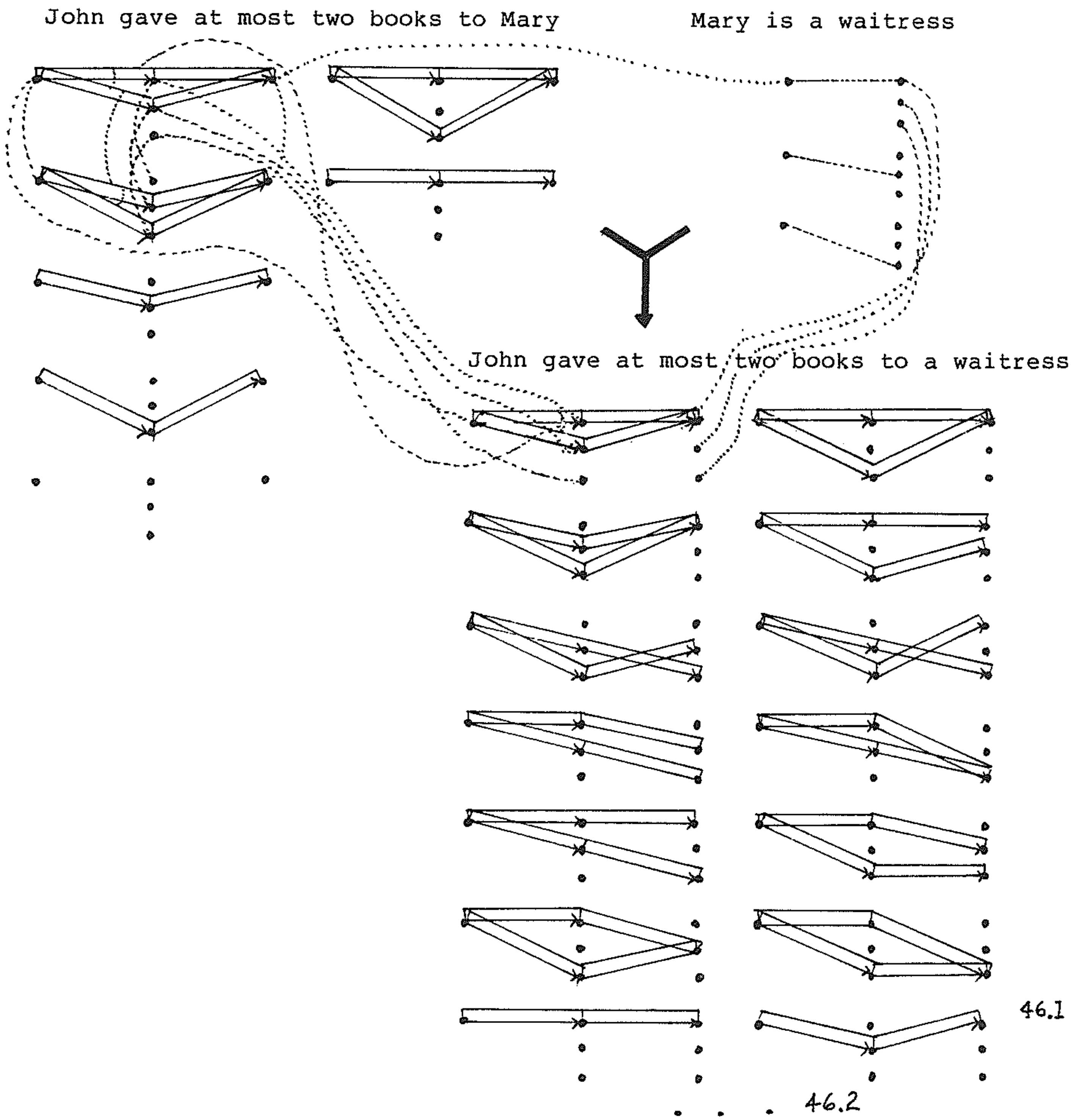
In the first example below, all EPDs are similar so that, as remarked in Section 1, entailment among EDs resolves to the set-theoretic inclusion of the EPDs in the entailing ED within the entailed ED.



Exactly
two
men
love
Mary
At least
two men
love
Mary
At least
one man
loves
Mary
At most
two men
love
Mary
Exactly
one man
loves
Mary
At most
one man
loves
Mary


At least two men love Mary
At least two men love
Mary is loved
At least one man loves

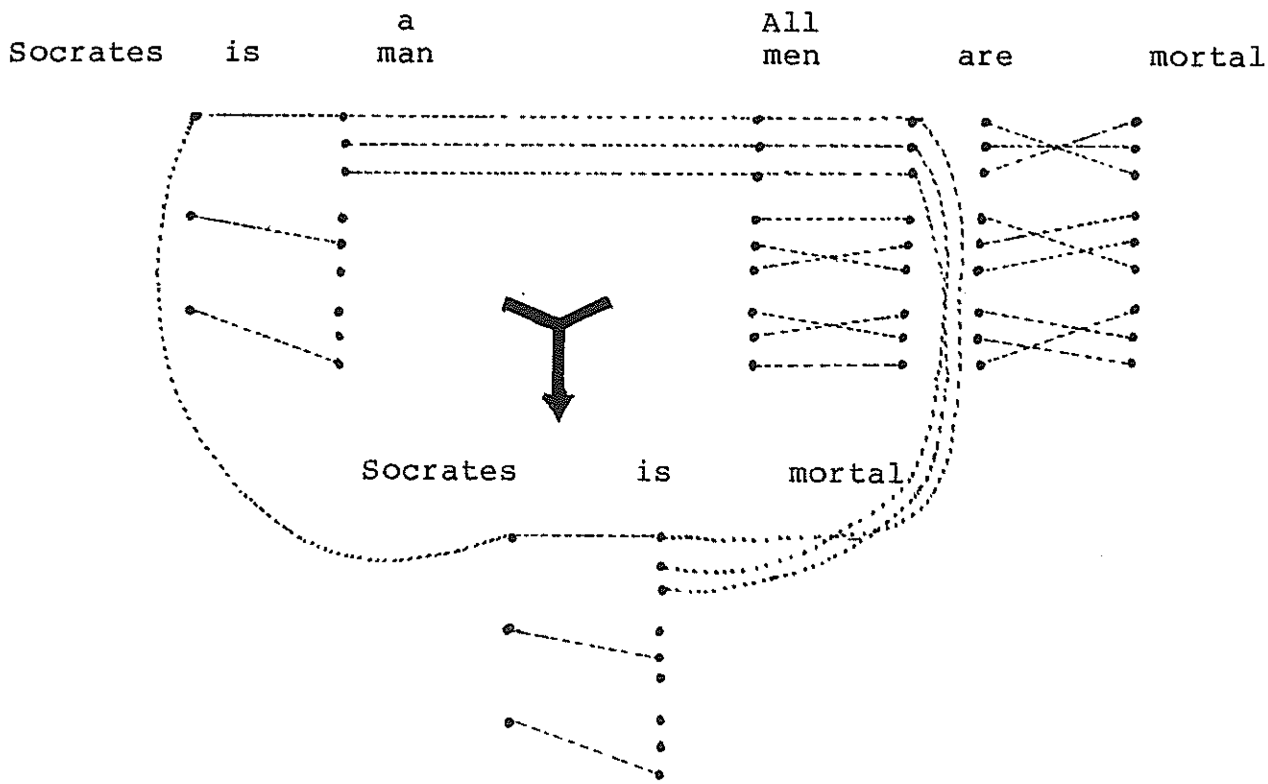
The preceding examples illustrated one-premise and two-premise entailment involving similar EDs. The following series of examples illustrates cases of two premise entailment involving non-similar and/or dot-path EDs.



46.1

46.2

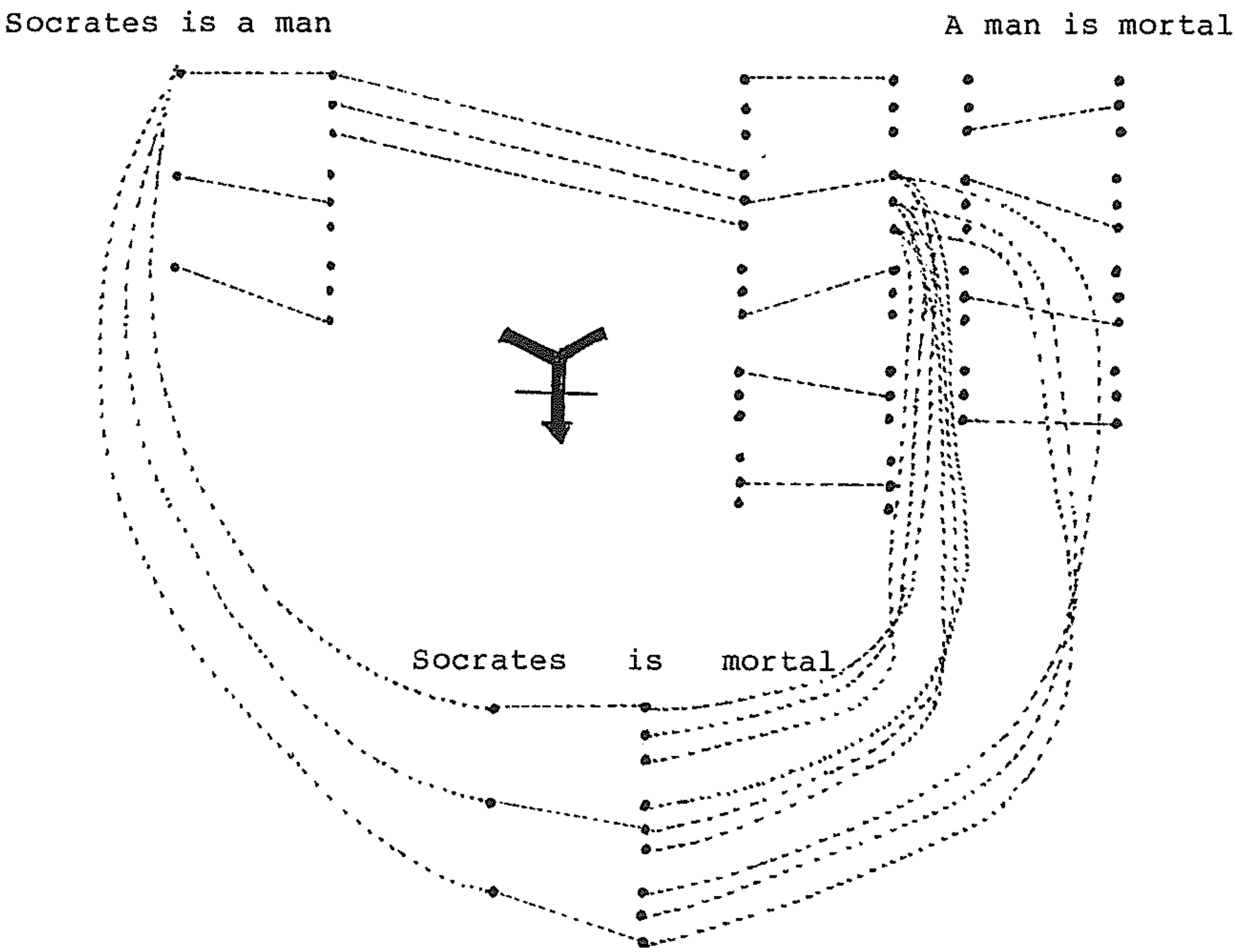
Note 46.1. Owing to the particular normal reading adopted of

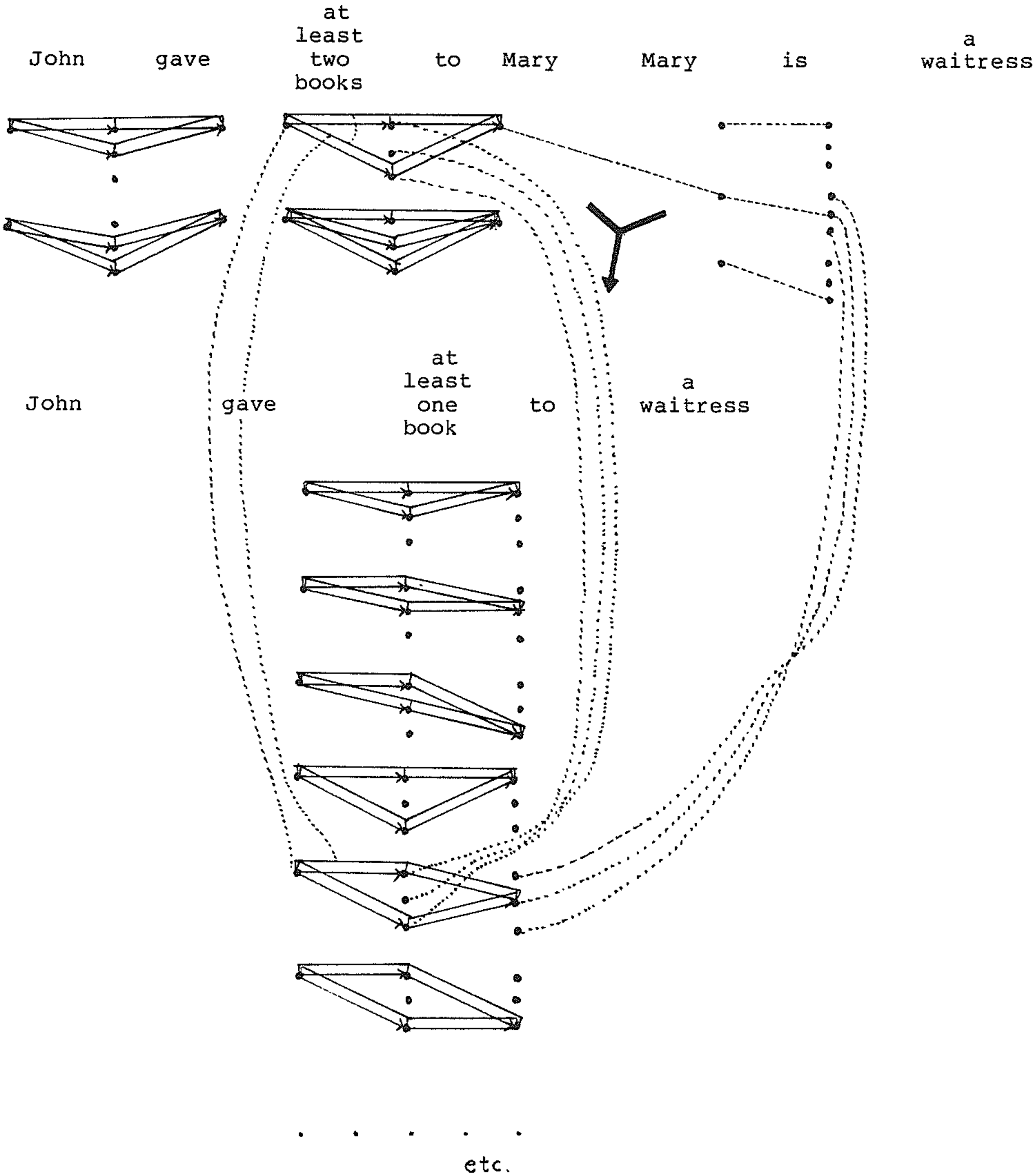


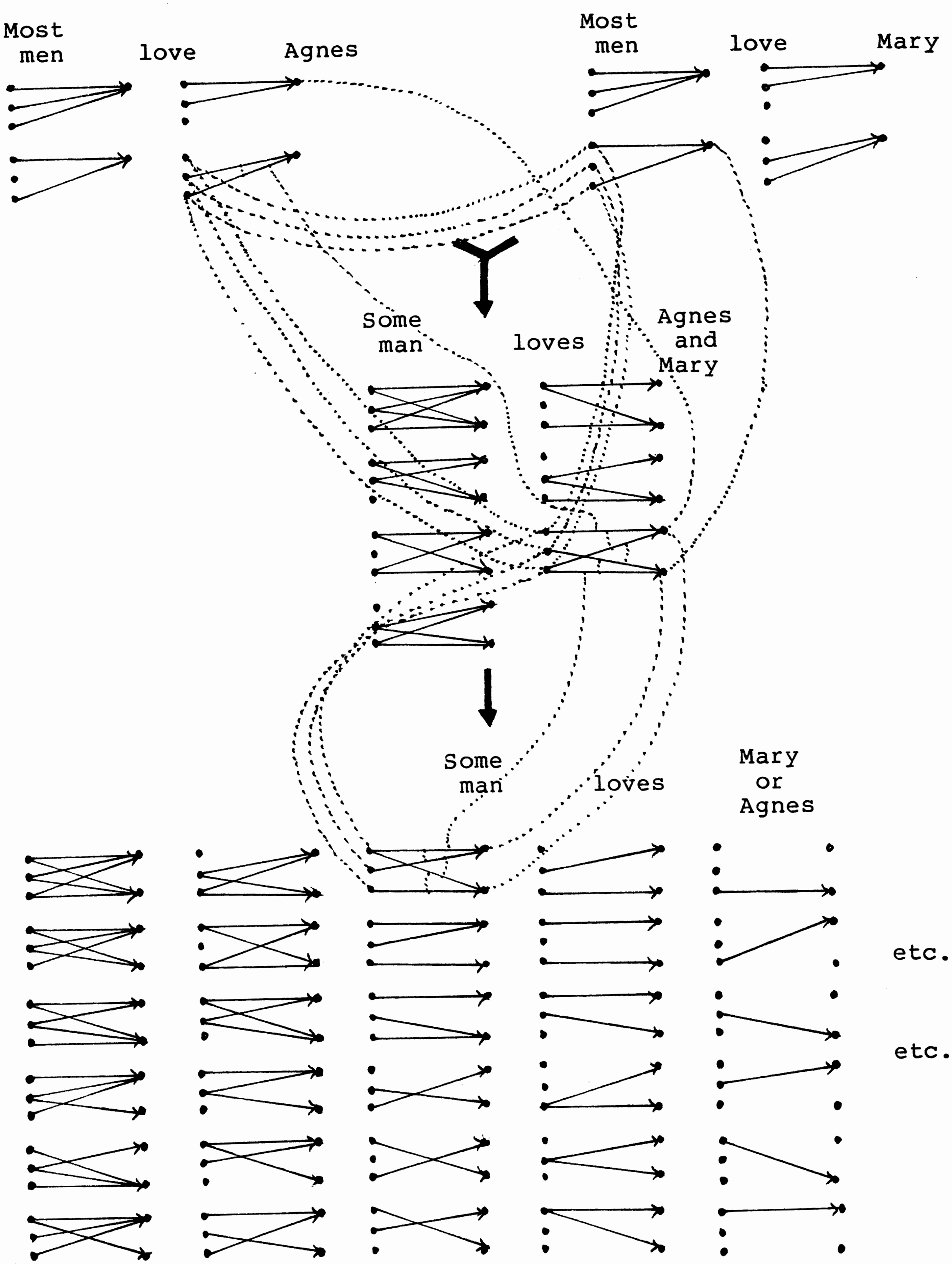
"John gave at most two books to a waitress," (see reading 15(b) of Note 39.2), all that is diagrammatically required here is that there is at least one point in the "waitress" point bank to which at most two points from the "book" point bank are connected by unbarred arrow traces.

Note 46.2. This ED contains $2^{3\times 3} - 1 = 255$ EPDs, of which we exhibit only 14 EPDs, the missing EPDs being indicated, as usual, by the use of dots . . . . We note, however, that the radical reduction of the full ED would contain only nine EPDs.

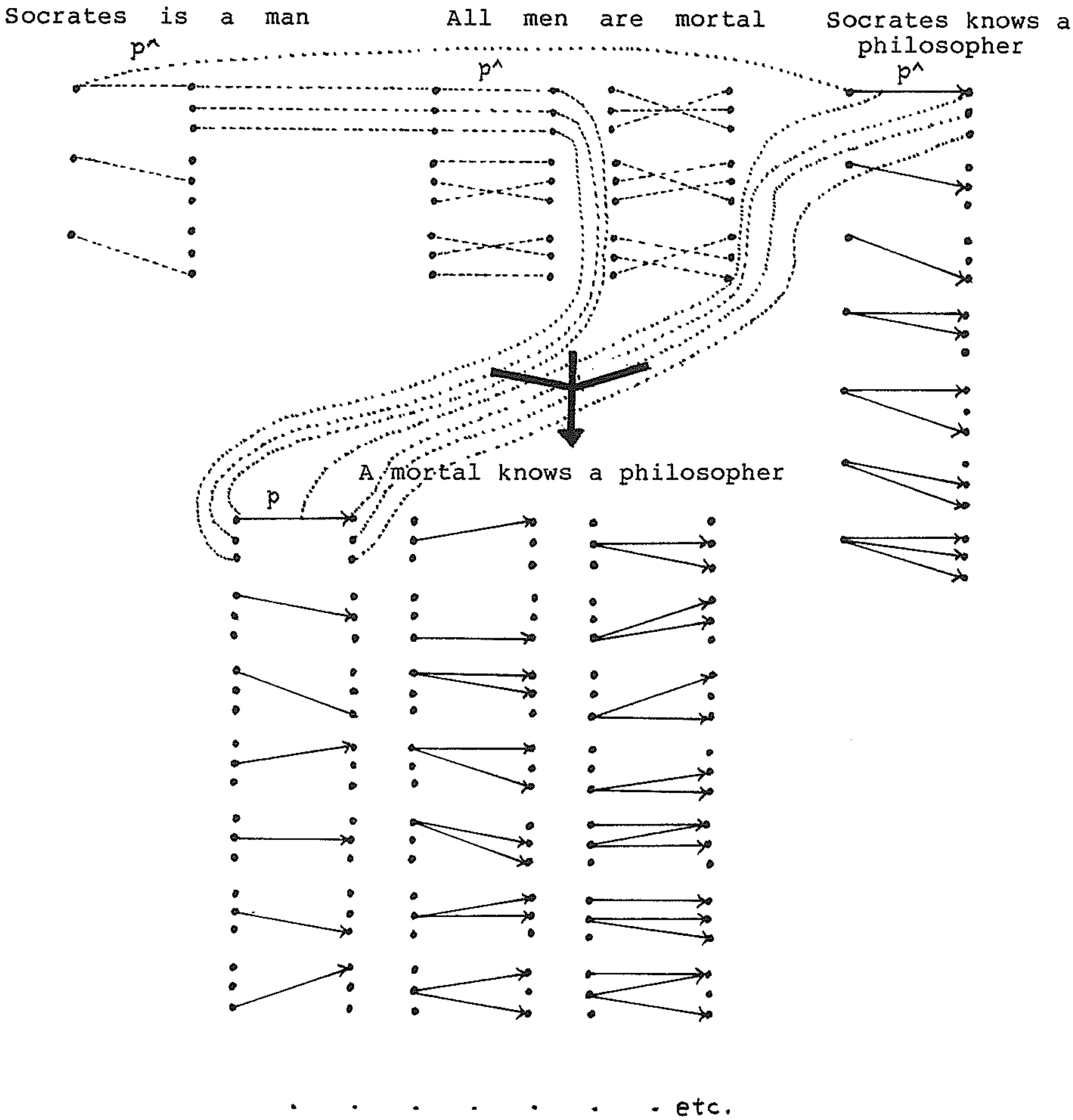
The following is a case of <u>invalid</u> entailment:




John
gave
at least two books
to
Mary
Mary
is
a waitress
John
gave
at least one book
to
a waitress
etc.

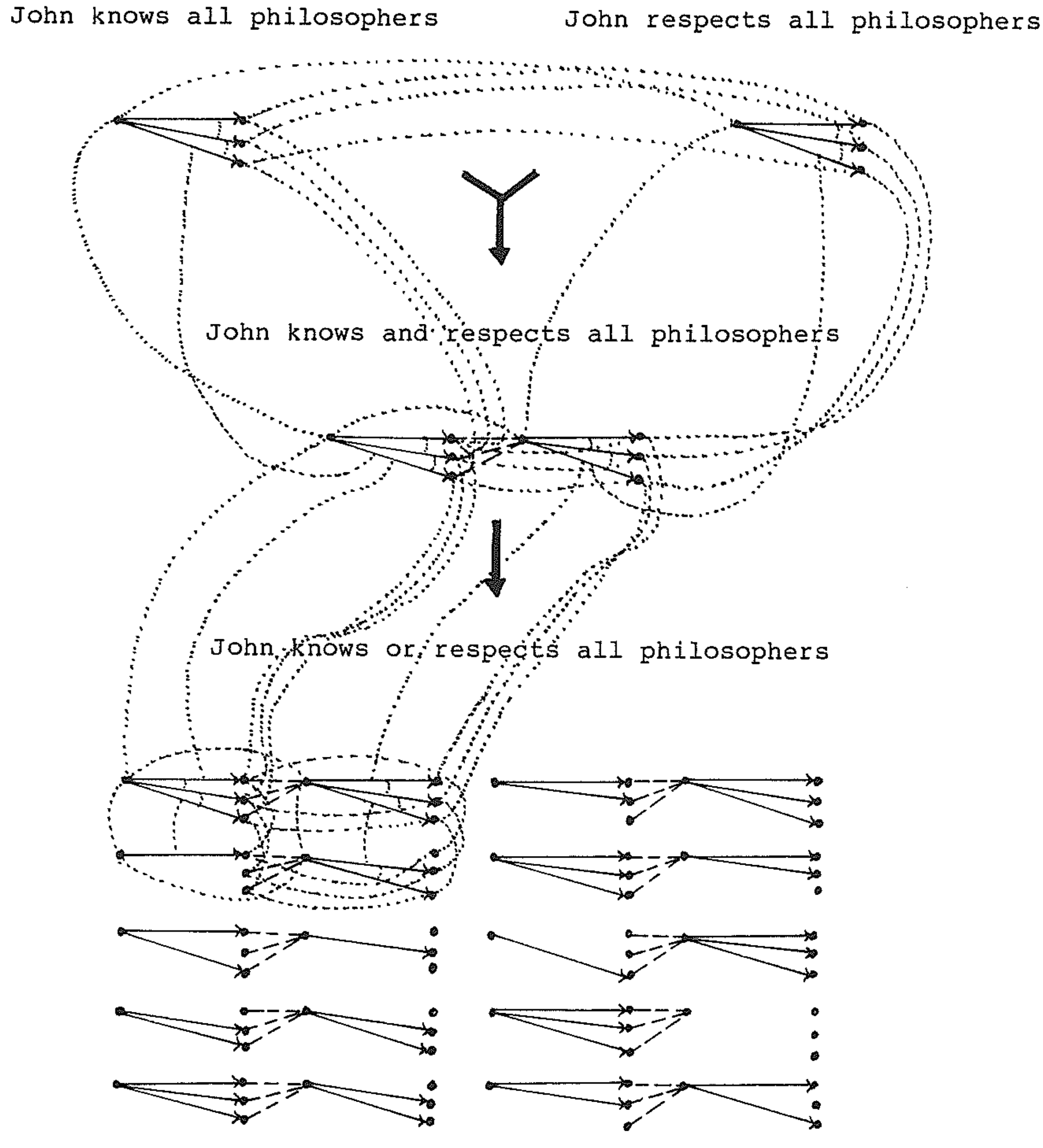

Most men
love
Agnes
Most men
love
Mary
Some man
loves
Agnes and Mary
Some man
loves
Mary or Agnes
etc.
etc.

The following illustrates a case of three-premise entailment. Generally speaking, arbitrary new event diagrams can be added to an existing semantic network, i.e., an RBSN, in the sense that their points and arrow traces are joined by dotted line links to those of the other event diagrams of the network. Then this augmented RBSN comprises a richer premise set from which entailments can be drawn.



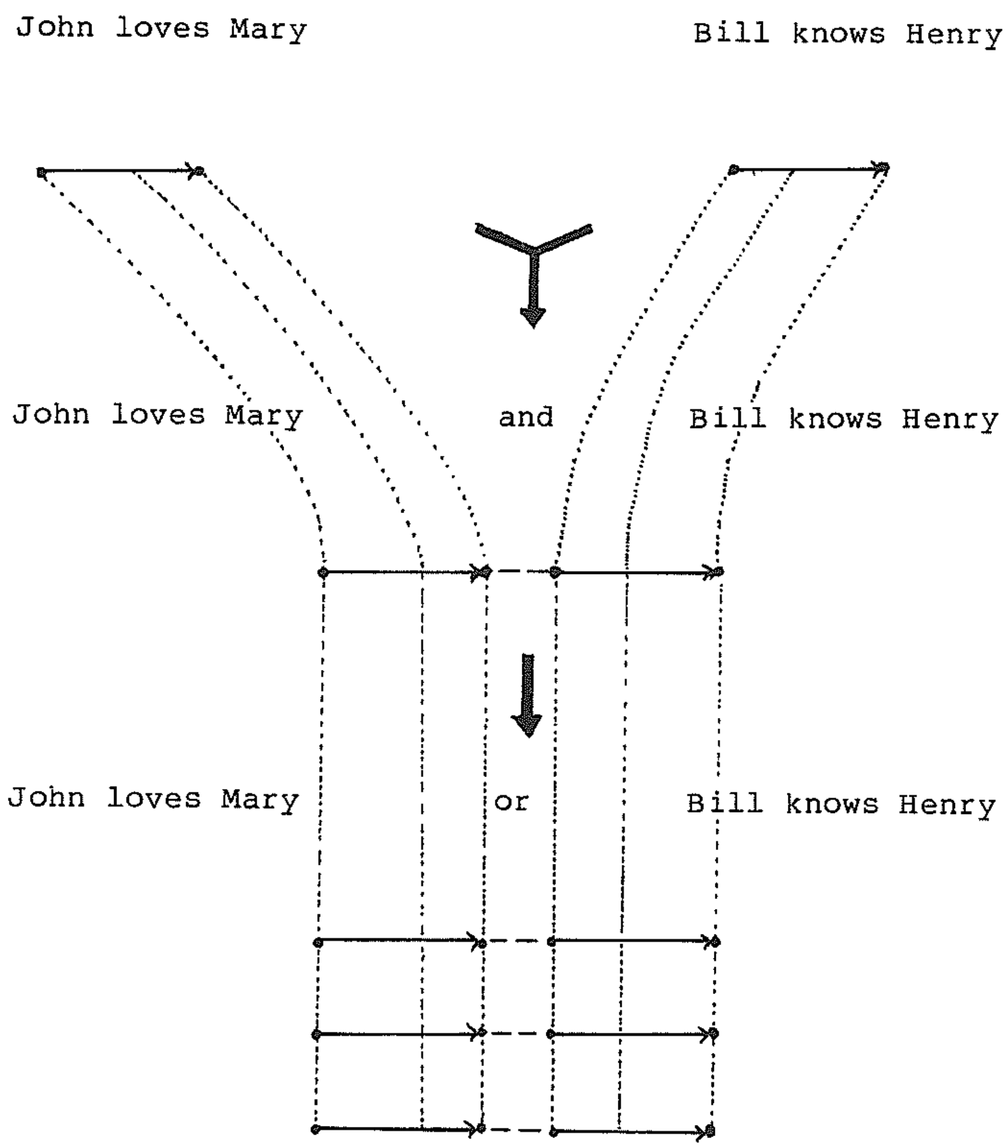
46.2

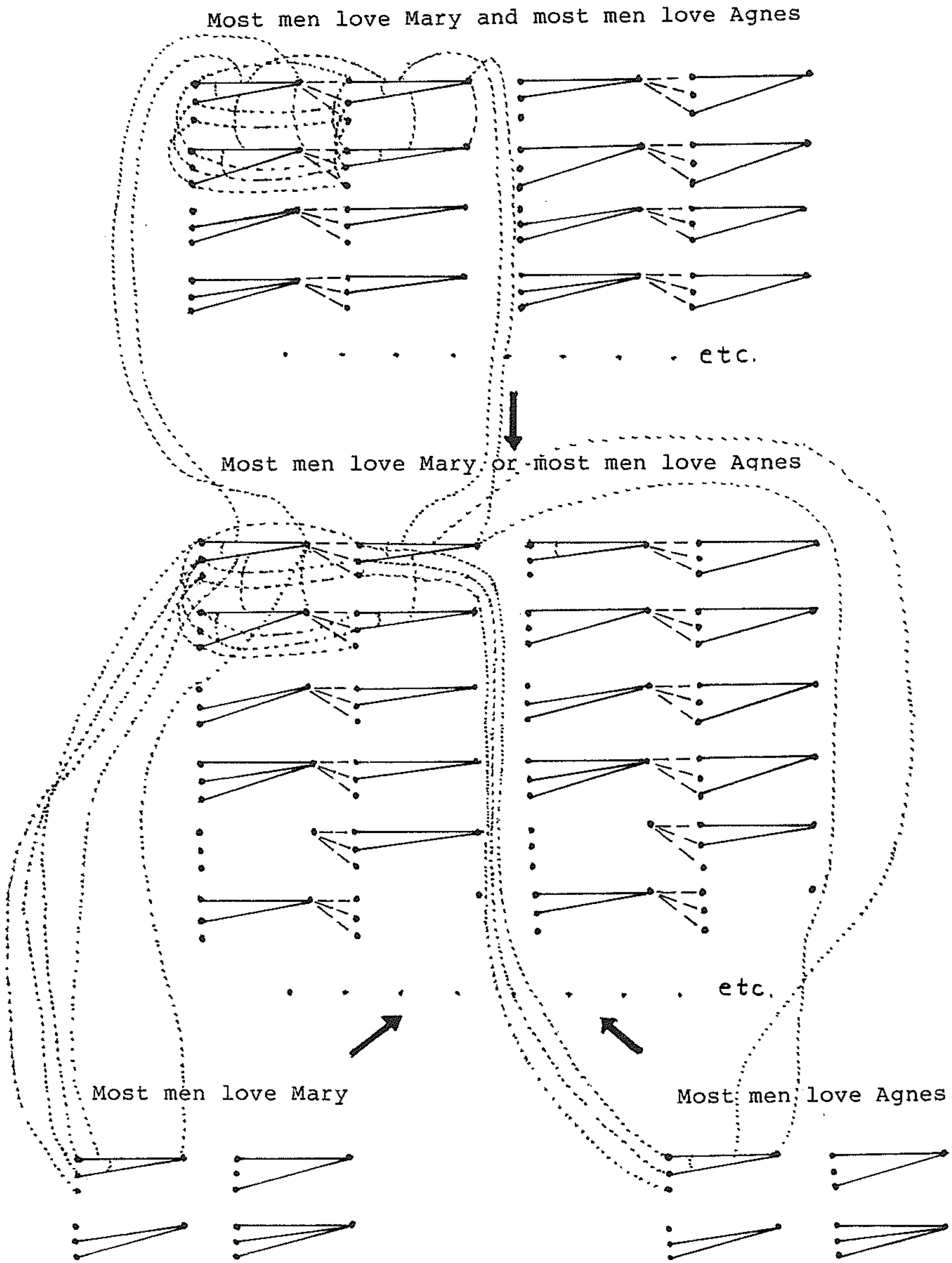
Note 46.2. The 2nd and 3rd man (represented by the 2nd and 3rd points in the "all men" point-bank) may or may not know a philosopher (represented by the 1st, 2nd, or 3rd points in the "a philosopher" point-bank). All possible signings of arrow paths joining the 2nd and 3rd man to any of the three philosophers will occur in the entailed ED with the same arrow path p , so that



Þ is a reduction set relative to that entailed ED. Hence all that is required is that this arrow path p be the resultant of some generalized arrow path of the three entailing EDs (labelled by p^).

Sentential entailment




Most men love Mary and most men love Agnes
etc.
Most men love Mary or most men love Agnes
etc.
Most men love Mary
Most men love Agnes

APPENDIX

In this Appendix we describe in somewhat more detail certain aspects of the structure of readings as developed in ATR, and the way that those aspects are represented in RBSNs. To keep this paper within reasonable limits our account of the structure of RBSNs is limited to those aspects of that structure that reflect only the surface organization of readings of natural language sentences. Accordingly, our description in this Appendix is restricted to an account of that surface organization, omitting consideration of the structure of readings of nonsentential word-strings. In particular, we do not discuss the internal semantic structure of readings of subsentential word-strings such as words, phrases, and clauses, nor of supersentential word-strings involving sequences of sentences, nor do we include descriptions of readings of special grammatical constructions such as modification, pronominal referencing, comparatives, modal and temporal adverbial constructions, etc.[47]

The general linguistic orientation taken in ATR is that a natural language word-string e is to be regarded as a "signal system" which, in conjunction with the context-of-utterance in which e is produced, signals one or more "normal" i.e., plausible readings of e of varying "degrees of normality", each reading of

Note 47. Our methods readily accommodate the treatment of these further constructions as well as the simpler ones described herein.

e constituting a complete formal description of a particular way of understanding e.

A reading of a natural language word-string e is a pair (e',s), where e' is a syntactic representation of e and s is a semantic theory that interprets e'.

The syntactic representation e' of e is an expression of a formal representation language L'. The structure of e' is referred to as the syntactic structure of the word-string e under the reading (e',s). The semantic theory s consists of (i) a family F_s of interpretations (D,f), where D is a universe of discourse and f is a denotation function which assigns to every expression b' of L' a set f(b') constructible from that universe of discourse, and (ii) a valuation function V_s that assigns a truth value (truth, falsehood, or nil) to every pair (e', (D,f)) consisting of a syntactic representation e' of a sentence e of L, and an interpretation $(D,f) \in F_s$.[48] For the purposes of this paper we adopt a simple version of the valuation function V_s which is such that $V_s(e',(D,f))$ = truth if and only if, $f(e') \neq \phi$ and, letting $e' = r^m(a_1,\ldots,a_m)$, we have that, for all $1 \leq i \leq m$, $\bigcup f(a_i) \neq \phi$.[49] The semantic structure of e' relative to the semantic theory s is the set of

Note 48. In ATR the semantic theory s also includes a third component, namely a set of binary relations on interpretations, which provides for the semantic treatment of modal operators in the manner first introduced by Kripke.

Note 49. This simple valuation function V is not the only one possible. We will not discuss other alternatives here, however, since such a discussion would require also an examination of the finer breakdown of L'-expressions given in ATR, but which goes beyond the scope of this paper. The simple valuation function V used here prohibits the possibility of EPDs with empty point banks. That is to say, the adoption of this valuation function is tantamount to the assumption that all sentences have "existential import" in the sense that, if $r^m(a_1,\ldots,a_m)$ were true in all sets $f(e')$, as (D,f) ranges over all interpretations of s. It is this aggregate structure that we refer to as the semantic structure of the word-string e under the reading (e',s).[49.1]

The Syntactic Structure of Natural Language Sentences. In the case where the word-string e L is a sentence, its syntactic representation $e' \in L'$ at the "surface organization" level consists of (a) an expression r^m of L', called an m-place relation expression which we can schematically write as $r(c_1,\ldots,c_m)_t$, composed of a base relation expression r of L' together with m expres-

some interpretation (D,f) under V, then for each $1 \leq i \leq m$, there would exist some elements in $\bigcup f(a_i)$.

Note 49.1 We can also distinguish another reasonable sense of semantic structure as "intrinsic": The intrinsic semantic structure of b' under the family F_s of interpretations is the common structural set-theoretic properties shared by all the sets $f(b')$, as (D,f) ranges over F_s such as, for example, being a set of subsets of the universe of discourse D, being a set of subsets of the universe of discourse that is closed under unions, intersections, or some other operation, being a set of n-tuples of elements of D, a set of n-tuples having some particular structure, etc. Such properties are fixed by the special semantic axioms defining that semantic theory, which axioms specify the common structural properties that given (logical) expressions $b' \in L'$ are assigned under all interpretations (D,f) of the semantic theory.

sions $c_1,\ldots,c_m$ of L', called case-expressions, together with an ordering[50] t on those case expressions called the case-ordering on $c_1,\ldots,c_m$, and (b) m further expressions $a_1,\ldots,a_m$ of L', called thing-expressions, together with two orderings p,q on $a_1,\ldots,a_m$, called respectively the relative scope and relative-place orderings on $a_1,\ldots,a_m$.[51] The case ordering t on the case expressions $c_1,\ldots,c_m$ determines the way that the relation-expression r^m is composed out of the base relation-expression r and those case-expressions $c_1,\ldots,c_m$ (different case orderings corresponding, very roughly, to the difference between those possible readings of "Every man loves some woman" reflected in the respective dominant readings of "Every man is such that he loves some woman" and "Every man is such that he is loved by some woman"); the relative-place ordering q on the m thing-expressions $a_1,\ldots,a_m$ determines the order in which these thing-expressions

Note 50. The notion of a reading presented in ATR is sufficiently flexible as to provide syntactic representations of meaningful natural language word-strings that mimic the surface organization of those word-strings as closely as possible, preserving, in particular, adjacency relationships among the parts of the word-string. Because of this and because we wish to have readings available for arbitrary sorts of meaningful word-strings or arbitrary natural languages, the syntactic representations adopted, particularly for sentences, include reference to the indicated orderings. To allow a closest possible relationship between syntactic and network representations, network representations of sentences include network analogues of such orderings.

Note 51. The displayed order indicated by the subscripts 1,...,m has no significance, and is simply an accidental consequence of writing the m expressions in a linear array. However, when these expressions are explicit rather than schematic (as here), for expository purposes, the order of display is written in such a way that those expressions among them that are signalled by explicit word-strings or word-string parts appear in the same order as exists among those signalling word-strings or word-string parts.

are to be taken relative to r^m in the sense that the thing-expression a_i is to occupy the j^{th} argument place of r^m (different relative-place orderings corresponding, very roughly, to the difference between those possible readings of "Every man loves some woman" reflected in the respective dominant readings of "Every man is such that he loves some woman" and "Some woman is such that she loves every man"); the relative-scope ordering p on the m thing-expressions $a_1,\ldots,a_m$ determines the order in which these m thing-expressions are to be taken relative to the scopes of their governing determiners (different relative-scope orderings corresponding, very roughly, to the difference between those possible readings of "Every man loves some woman" reflected in the respective dominant readings of "Every man is such that he loves some woman", and "Some woman is such that every man loves her".)[52] With these understandings of the intended meanings of $r, c_1,\ldots,c_m$, $a_1,\ldots,a_m$, t, p, and q, we can schematically indicate the syntactic structure of the natural language sentence e under the reading (e',s) as $[r(c_1,\ldots,c_m)]_t\ (a_1,\ldots,a_m)\ p,q$, or, when not exhibting the case-expressions of r^m, simply as $r^m(a_1,\ldots,a_m)p,q$.

Note 52. The need to explicitly recognize such ordering relations as contributing to the ultimate grammatical organization ofnatural language sentences is masked somewhat by the fact that, in syntactic representations of most common natural language sentences, the ordering imposed by each ofthese relations can be taken as coinciding with the say, left to right, order of occurrence of those word-strings or word-string parts tha comprise explicit markers within a natural language sentence signalling those case-expressions and thing-expressions in those syntactic representations. But in the general case, their coinciding with such occurrence orders (or with each other, for that matter) cannot be assumed, so that relativization to these orderings is required.

The Semantic Structure of Natural Language Sentences. The semantic theory s is such that for each constituent interpretation (D,f) of s, (D,f) assigns to each m-place relation-expression r^m a set $f(r^m)$ of ordered m-tuples of elements of D, and to each thing-expression a_i, $1 \leq i \leq m$, (D,f) assigns a set of subsets of D. The internal structure of these sets of subsets varies according to the determiners entering into the syntactic structure of the thing-expression to which the set is being assigned as denotation. Broadly speaking, for each semantic theory s, we distinguish two types of thing-expressions, those that are non-limiting under s, and those that are limiting under s. The semantic structure of the denotations of non-limiting thing-expressions under s can be formulated as follows: A thing-expression a is non-limiting under s if and only if, for all interpretations (D,f) of s, $\bigcup f(a) \neq \phi$ and f(a) is closed under supersets in the sense that if $x \in f(a)$ and $x \subseteq y \subseteq \bigcup f(a)$, then $y \in f(a)$. Intuitively, non-limiting thing-expressions represent noun phrases in English, say, that impose no upper limits on their denotations, such as "John", "some man", "all men", "at least two men", and so on, in the sense that if something were true of some particular entity John, some men, all men etc., it would not prohibit its being true of further entities (if they existed) beyond John, beyond some (particular) man, beyond all men, beyong at least two men, and so on. Non-limiting thing-expressions are distinguished from limiting thing-expression, and represent noun phrases which do impose upper limits on their denotations, like "at most John", "at most two men", "exactly two men" "all but two men", "all men except John" and so on, in the sense that if something were true of *at most John*, *at most two men*, *exactly two men*, *all but two men, or all men except John*, and so on, it could not also be true of any entities beyond *John*, beyond *at most two men*, beyond *exactly two men*, *all but two men*, *all men except John*, and so on. A sentence representation $r^m(a_1,\ldots,a_m)$ is then said to be *non-limiting under s* if and only if each of its major thing-expressions $a_1,\ldots,a_m)$ is non-limiting under s; otherwise $r^m(a_1,\ldots,a_m)$ is said to be *limiting under s*.

We define the denotation of a sentence representation $r^m(a_1,\ldots,a_m)$ p,q under an interpretation (D,f) of s as a set $f[\, r^m(a_1,\ldots,a_m)]$ p,q called an *event*, which is wholly determined by $f(r^m), f(a_1,\ldots,f(a_m)$, and the orderings p and q on $a_1,\ldots,a_m$, as follows:

Let $B_1,\ldots,B_m \subseteq PD$. A *chain function on the sequence $(B_1,\ldots,B_m)$* is a function g which assigns, for every $1 \leqslant i \leqslant m-1$ and for every $y \in \bigcup B_i$, a set $g(i,y) \in B_{i+1}$.[53]

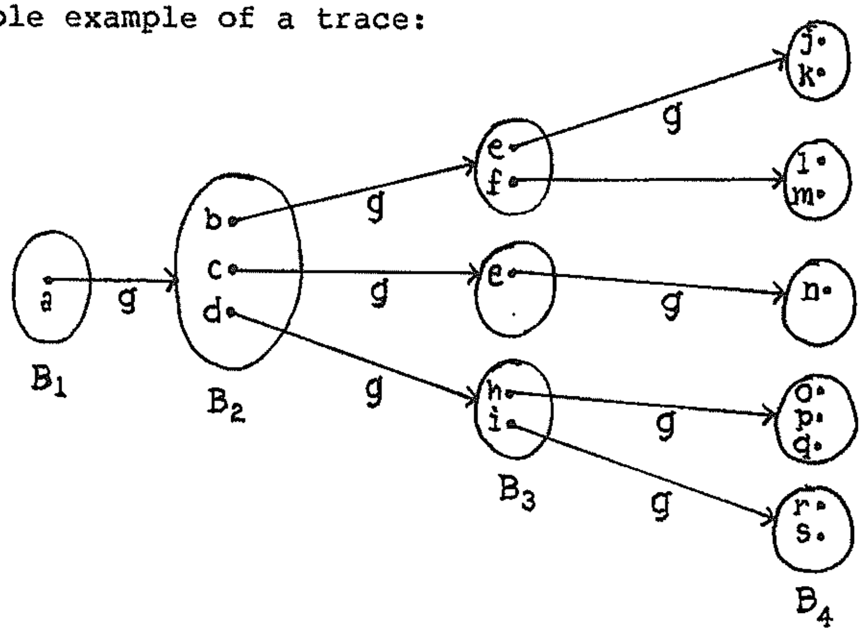
Let $B_1,\ldots,B_m \subseteq PD$, let q be a permutation on $\{1,\ldots,m\}$, and let g be a chain function on the sequence $(B_1,\ldots,B_m)$. Then *the trace of g through $(B_1,\ldots,B_m)$ with respect to q* is the set:

$$\{ (z_{q^{-1}(1)},\ldots,z_{q^{-1}(m)}) \in D^m \mid \text{for some } x_1 \in B_1 \ ,\ z_1 \in x_1 \text{ and } z_2 \in q(1,z_1) \text{ and } z_3 \in g(2,z_2) \text{ and } \ldots \text{ and } z_m \in g(m-1\ ,\ z_{m-1}) \}.$$

For the special case that each of p, q is the identity permutation, then the trace of g through $(B_1,\ldots,B_m)$ with respect to q is the set:

Note 53. We introduce the parameter i in g(i,y) to accommodate the case where y occurs in both B_i and B_j for $i \neq j$.

$\{(z_1,\ldots,z_m) \in D^m \mid$ for some $x_1 \in B_1$, $z_1 \in x_1$ and $z_2 \in g(1,z_1)$ and $z_3 \in g(2,z_2)$ and ... and $z_m \in g(m-1, z_{m-1})\}$. Let us consider a simple example of a trace:



The trace of g through (B_1, B_2, B_3, B_4)

= $\{(a,b,e,j)$, (a,b,e,k) , (a,b,f,l) , (a,b,f,m), (a,c,e,n) , (a,d,h,o) , (a,d,h,p) , (a,d,h,q), (a,d,i,r) , $(a,d,i,s)\}$

The <u>positive relational structure</u> of $r^m(a_1,\ldots,a_m)p,q$ under (D,f) is the set $P_{(D,f)}\ [r^m(a_1,\ldots,a_m)p,q]$ =

$$f(r^m) \cap [\bigcup f(a_{q-1(1)}) \times \ldots \times \bigcup f(a_{q-1(m)})]$$

Note that if any of $B_1,\ldots,B_m$ is $\{\phi\}$, then a chain function can be defined on the non-empty sets among $B_1,\ldots,B_m$, and for every such chain function g the trace of g through $B_1,\ldots,B_m$ is the empty set.

Then we define

$$f[r^m(a_1,\ldots,a_m)p,q] = \begin{cases} (f[r^m], \{P_{(D,f)}[r^m(a_1,\ldots,a_m)p,q]\}), \text{ if } P_{(D,f)} [r^m(a_1,\ldots,a_m)p,q] \text{ is consistent with the determiner structure of } r^m(a_1,\ldots,a_m)\ p,q \text{ under } (D,f) \text{ (in the sense that there are non-empty subsets } B_1 \subseteq f(a_{p-1(1)}), \ldots, B_m \subseteq f(a_{p-1(m)}) \text{ and there is a chain function } g \text{ on } (B_1,\ldots,B_m) \text{ such that the trace of } g \text{ through } (B_1,\ldots,B_m) \text{ with respect to } q \text{ is identical with } P_{(D,f)} [r^m(a_1,\ldots,a_m)p,q]); \\ \text{and } = \phi, \text{ otherwise.} \end{cases}$$

That is to say, if (if r is the major relation-expression of the sentence b, and) the positive relational structure $P_{(D,f)}$ [b] under (D,f) of an L-sentence b is consistent with the determiner structure of b under (D,f), then f[b] = $\{(f[r^m], \{P_{(D,f)}[b]\})\}$; otherwise $f\{b\} = \phi$.

It can be easily proven that every chain function g on $B_1,\ldots,B_m$ yields a non-empty trace on $(B_1,\ldots,B_m)$ with respect to q if and only if none of $B_1,\ldots,B_m = \{\phi\}$. Therefore, if $P_{(D,f)}$ $(r^m(a_1,\ldots,a_m)p,q)$ is consistent with the determiner structure of $r^m(a_1,\ldots,a_m)p,q$ under (D,f) relative to $B_1,\ldots,B_m \neq \{\phi\}$, then $P_{(D,f)}$ $[r^m(a_1,\ldots,a_m)p,q] \neq \phi$. Furthermore, it can also be verified that if $f[r^m(a_1,\ldots,a_m)p,q] \neq \phi$ while $P_{(D,f)}$ $[r^m(a_1,\ldots,a_m)$ $p,q] = \phi$, then there is some chain function g on $(B_1,\ldots,B_m)$, where $B_1,\ldots,B_m$ are non-empty sets such that $B_1 \subseteq f(a_{p-1(1)}),\ldots,$

$B_m \subseteq f(a_{p-1(m)})$, such that the trace of g through $(B_1,\ldots,B_m)$ $= \phi$, which is to say (by the above observation), none of $B_1,\ldots,B_m = \{\phi\}$.

By the above definition, if $f[r^m(a_1,\ldots,a_m)\ p,q] \neq \phi$, then $f[r^m(a_1,\ldots,a_m)p,q] = \{\{P_{(D,f)}[r^m(a_1,\ldots,a_m)p,q]\}\}$. In the case that $f[r^m(a_1,\ldots,a_m)p,q] \neq \phi$, we call the pair $(f[r^m], P_{(D,f)}[r^m(a_1,\ldots,a_m)p,q])$ <u>the event particular corresponding to $r^m(a_1,\ldots,a_m)\ p,q$ relative to (D,f)</u>. Also, we define the <u>event corresponding to $r^m(a_1,\ldots,a_m)p,q$ relative to the semantic theory $S = (F_S,V_S)$</u> as the set of all event particulars corresponding to $r^m(a_1,\ldots,a_m)p,q$ relative to (D,f), as (D,f) ranges over F_S, and designate this event by $E_S[r^m(a_1,\ldots,a_m)p,q]$. It is also convenient to introduce a notation for the event particular corresponding to $r^m(a_1,\ldots,a_m)p,q$ relative to (D,f) in terms of f, namely as follows: $f[r^m(a_1,\ldots,a_m)p,q]^{\vee} = (f[r^m], P_{(D,f)}[r^m(a_1,\ldots,a_m)p,q])$ if $f[r^m(a_1,\ldots,a_m)p,q] \neq \phi$, and is undefined if $f[r^m(a_1,\ldots,a_m)p,q] = \phi$.

By the above definition of $f[r^m(a_1,\ldots,a_m)p,q]$ we note that the structure of the event particular $f[r^m(a_1,\ldots,a_m)p,q]^{\vee}$ is wholly determined by the denotations $f(r^m)$, $f(e_1),\ldots,f(a_m)$, and the orderings p,q. We have already remarked that we are concerned in this chapter only with the <u>surface organization</u> of the semantic structure of sentences: in the present context this means that we are concerned to depict only those aspects of the semantic structure of sentences that are determined by the denotations of its major relation-expression r^m under (D,f) and of its major thing-expressions $a_1,\ldots,a_m$ under (D,f), rather than the manner in which those denotations are determined by the denotations of their proper syntactic sub-expression. Accordingly, the notions of semantic structure to follow are defined only for sentences.

The semantic structure of $r^m(a_1,\ldots,a_m)$ p,q relative to the semantic theory $s = (F_s V_s)$ is identified with the event corresponding to $r^m(a_1,\ldots,a_m)$ p,q relative to s: E_s [$r^m(a_1,\ldots,a_m)$ p,q]. It is our intent that this semantic structure be graphically depicted by an event diagram associated with $r^m(a_1,\ldots,a_m)$ p,q relative to the semantic theory s. We will shortly indicate how event diagrams are obtained from events.

We note that if the semantic theory s is unconstrained, the semantic structure of $r^m(a_1,\ldots,a_m)$p,q relative to s is not readily conceptualizable inasmuch as its member event particulars are, in general, widely variable, consequently, the diagrammatic structure of the event diagram associated with it is not readily visualizable. However, we can impose suitable constraints on the semantic theory s to yield a semantic sub-theory of s which is sufficiently uniform as to render the notion of semantic structure both conceptually and visually coherent.

The requisite uniformity is obtained by: (i) constraining cardinalities of the denotations of thing-expressions via an index function h, to be the same across all interpretations (D,f) of the semantic theory s and (ii) constraining the constituent elements of those denotations so that the elements belonging to (the unions of) those denotations are identical.